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Topological charge pumping of cold atoms

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ETH Zurich

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Matthias Troyer

Xi Dai

Plan

Topological charge pumping in a 1D optical lattice

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Topological charge pumping in a 1D optical lattice

Measure topological index of 2D optical lattices

Pumps

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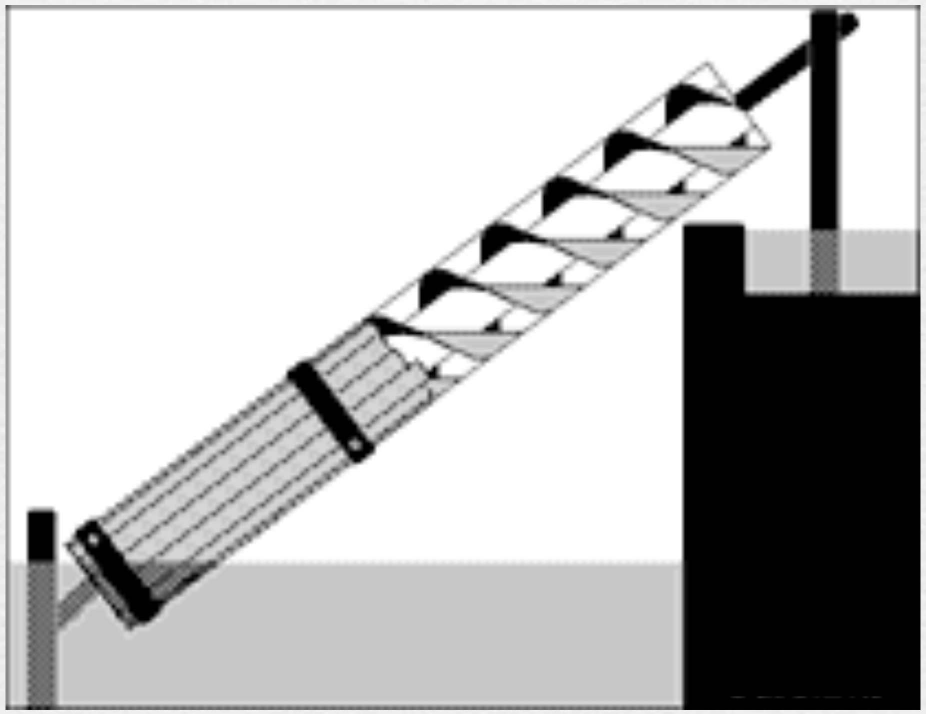

Archimedes' screw ~250 BC

Pumps

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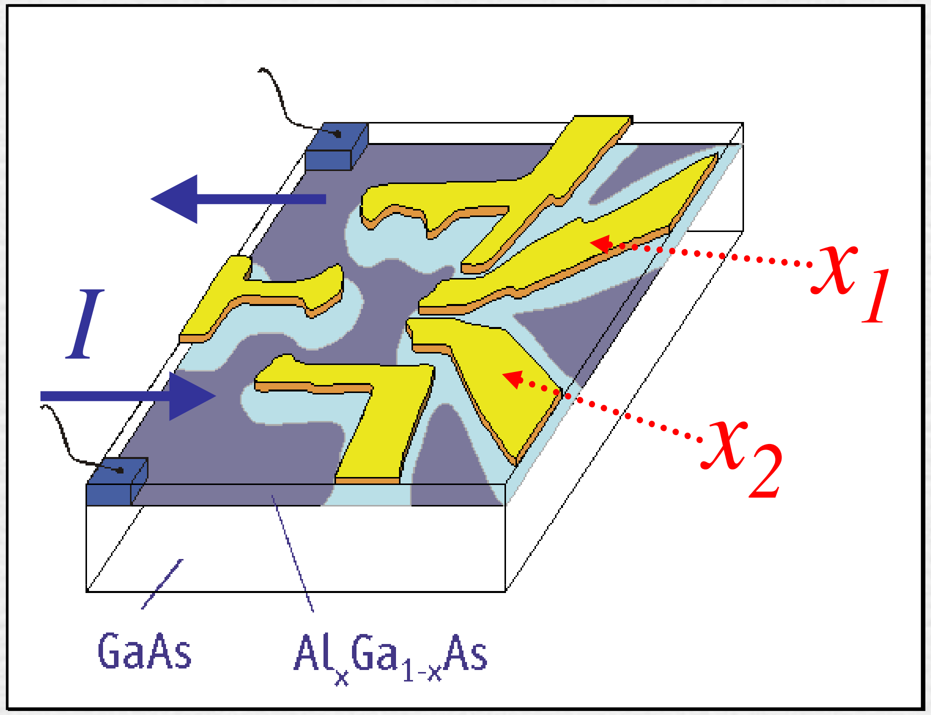
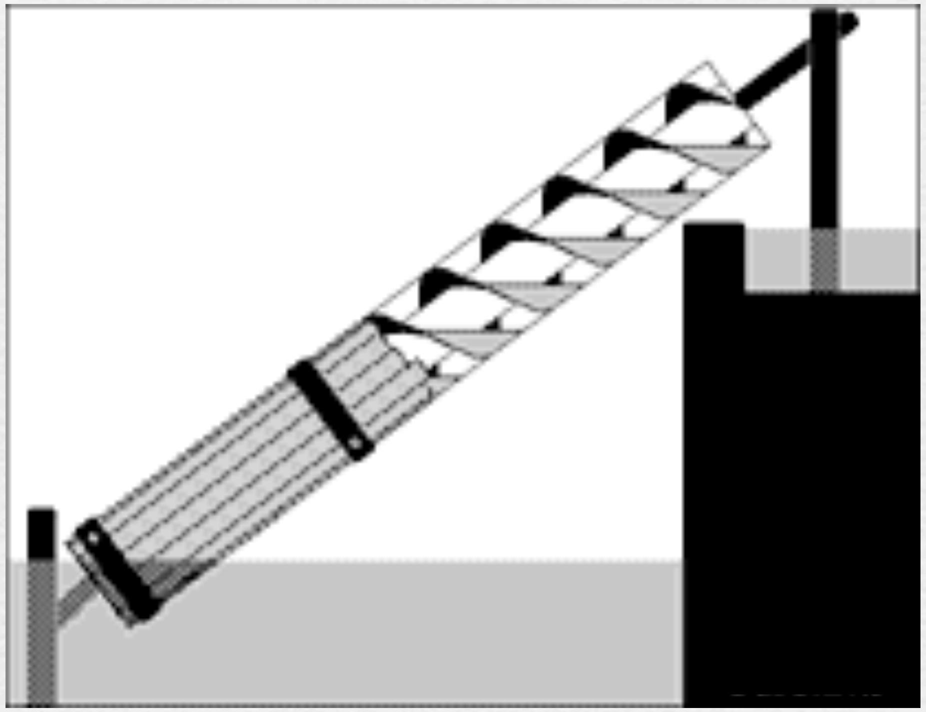


Switkes *et al* 1999

Buttiker, Brouwer, Zhou, Spivak, Altshuler ...



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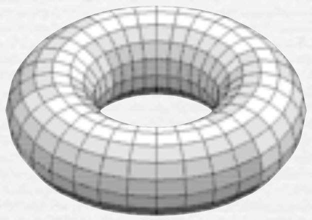
Topological pump

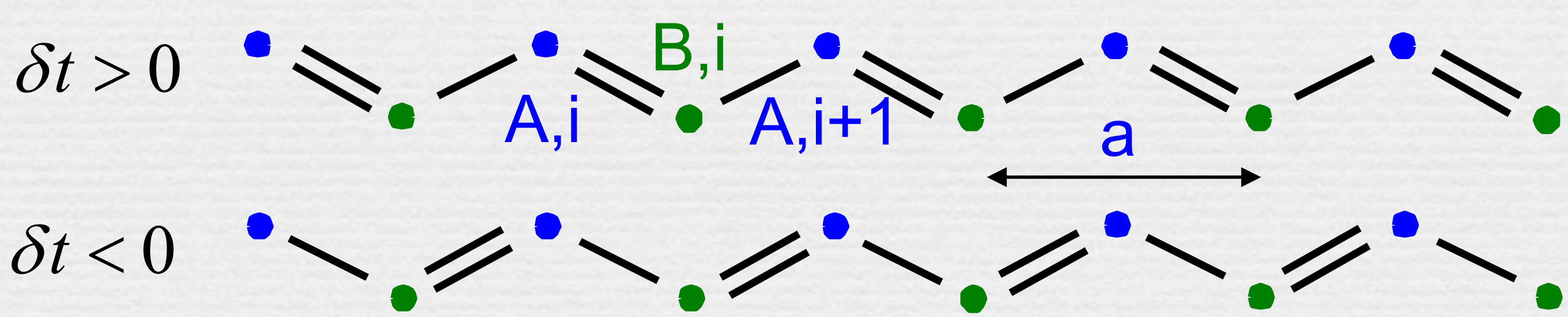


A device transfers **quantized charge** in each pumping cycle.

Thouless 1983

$$H = \sum_i (t+\delta t) c^\dagger_{Ai} c_{Bi} + (t-\delta t) c^\dagger_{Ai+1} c_{Bi} + H.c.$$

Su, Schrieffer, Heeger, 1979

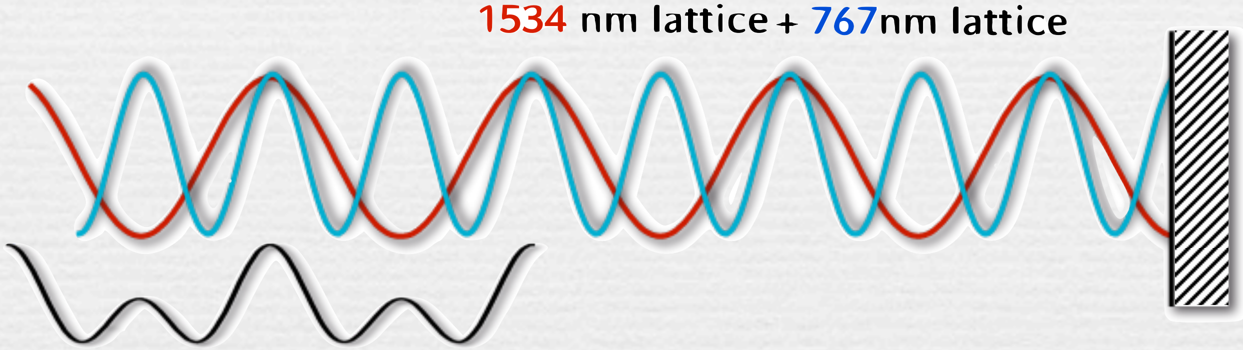


- Current flows in an insulating state
- No dissipation!
- Dynamical analog of quantum Hall effect

Experimental progresses

Optical Superlattice

Fölling *et al*, Atala *et al*

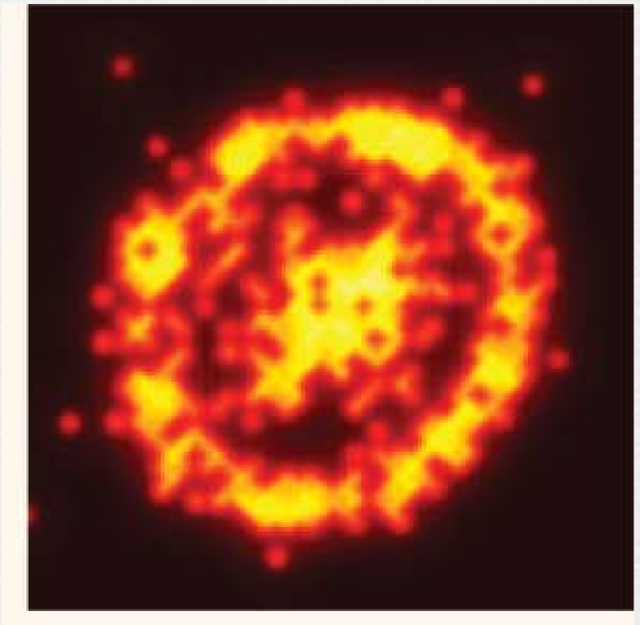


$$V_{\mathrm{OL}}(x) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \varphi\right)$$

Full (independent) dynamical control over V_1, V_2 and φ

in-situ imaging

Gemelke, *et al*, Sherson *et al*, Bakr *et al*



Allows to measure exact quantization of pumped charge

1D pumping lattices

$$V_{\mathrm{OL}}(x,t) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

$$V_1 = V_2$$

1D pumping lattices

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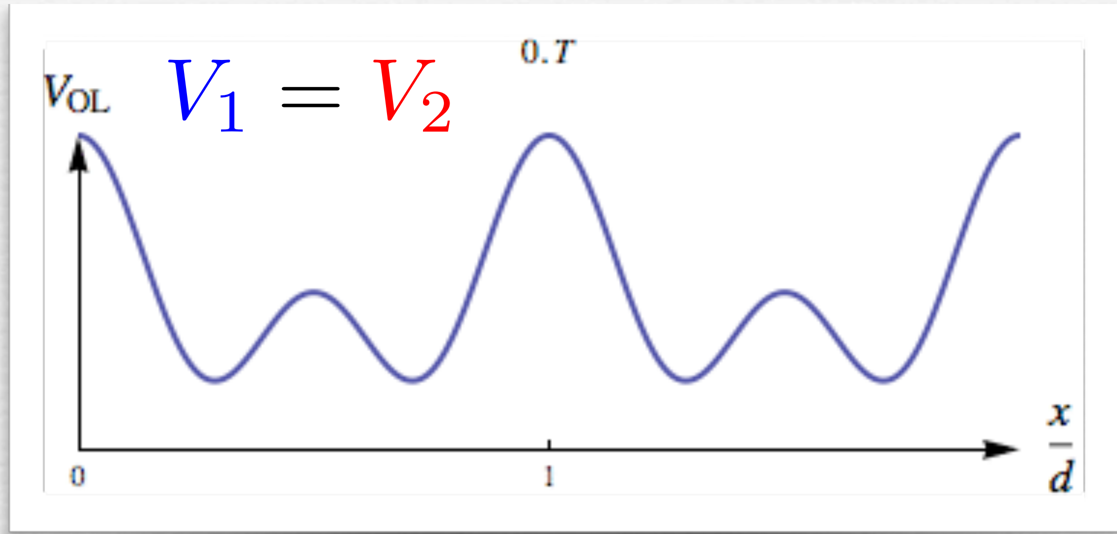

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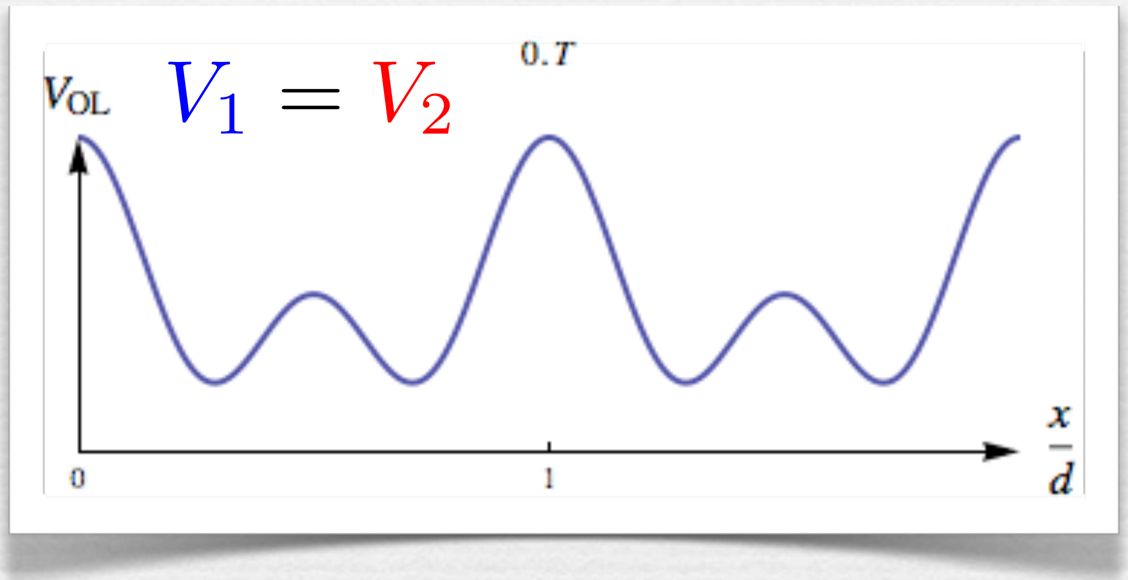

0 A ═ B — A ═ B

Su, Schrieffer, Heeger, 1979

T/2 A — B ═ A — B

1D pumping lattices

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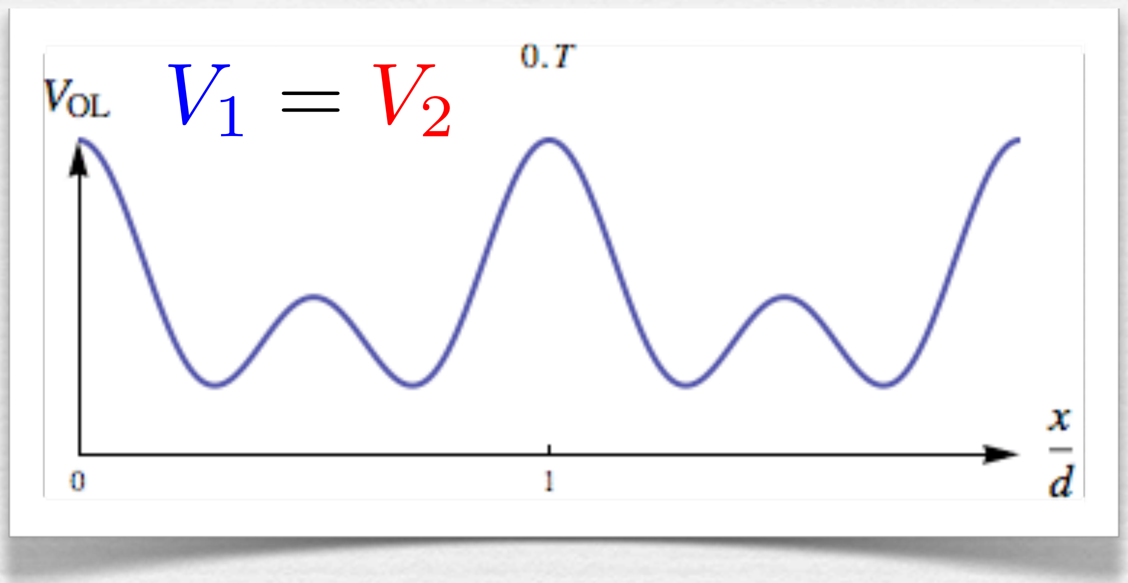


0 A ═══ B ─── A ═══ B

T/4 A - - - - - B - - - - - A - - - - - B

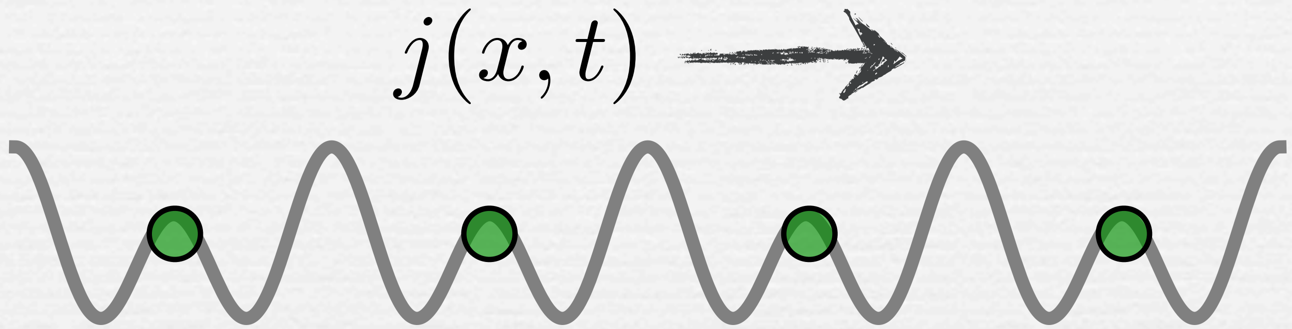
T/2 A ─── B ═══ A ─── B

3T/4 A - - - - - B - - - - - A - - - - - B

Su, Schrieffer, Heeger, 1979

Rice, Mele, 1982

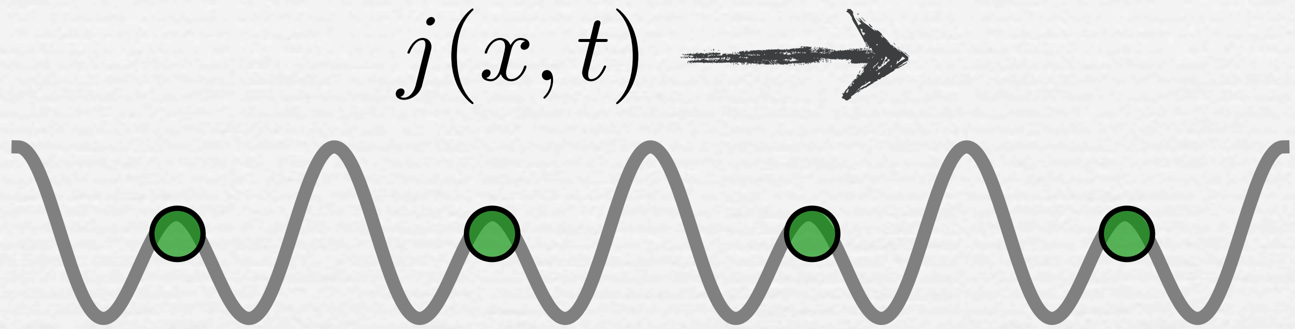
Pumping dynamics

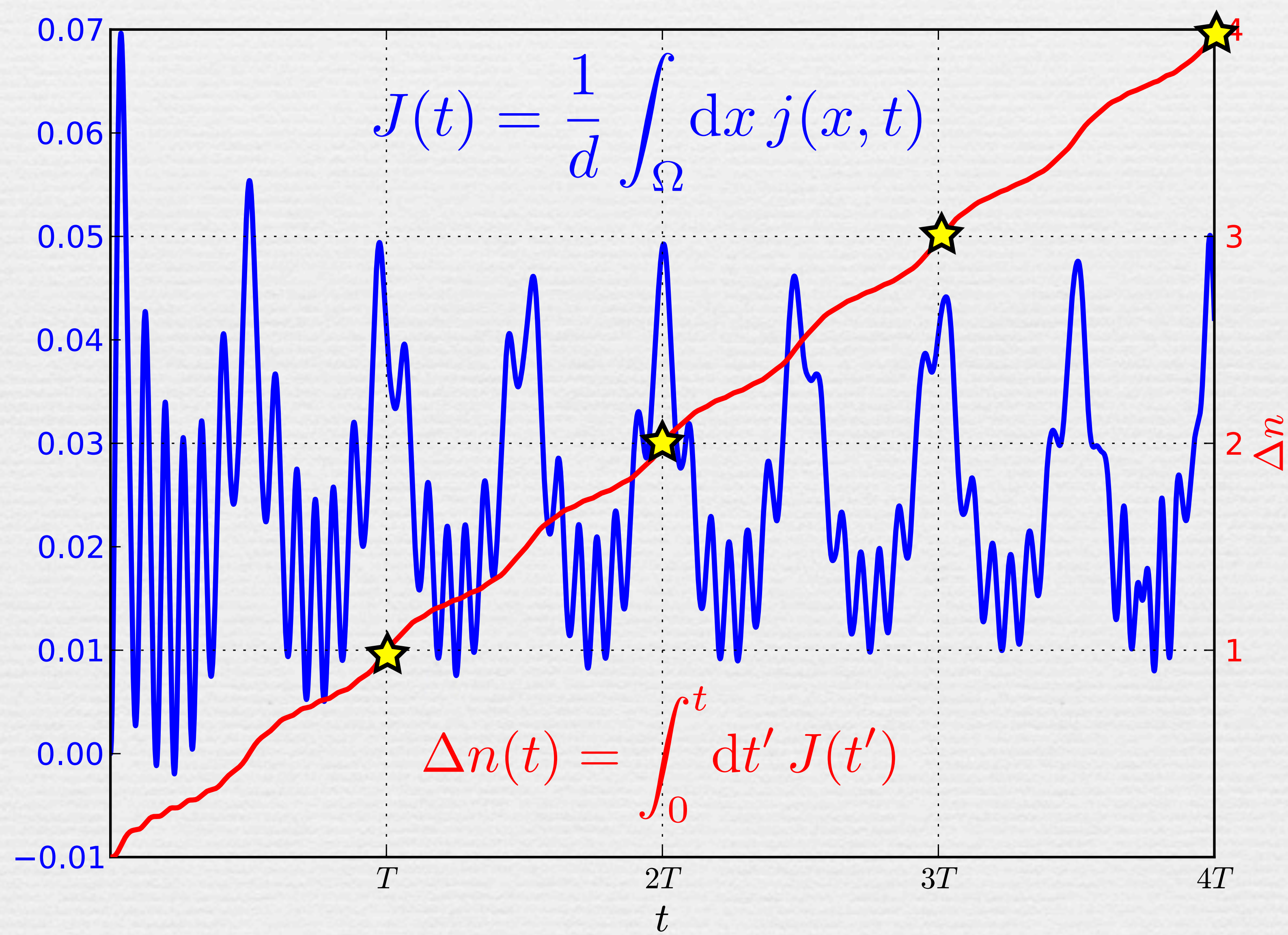


$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

$$i\frac{\partial}{\partial t}|\Psi\rangle = H(x,t)|\Psi\rangle$$

Pumping dynamics



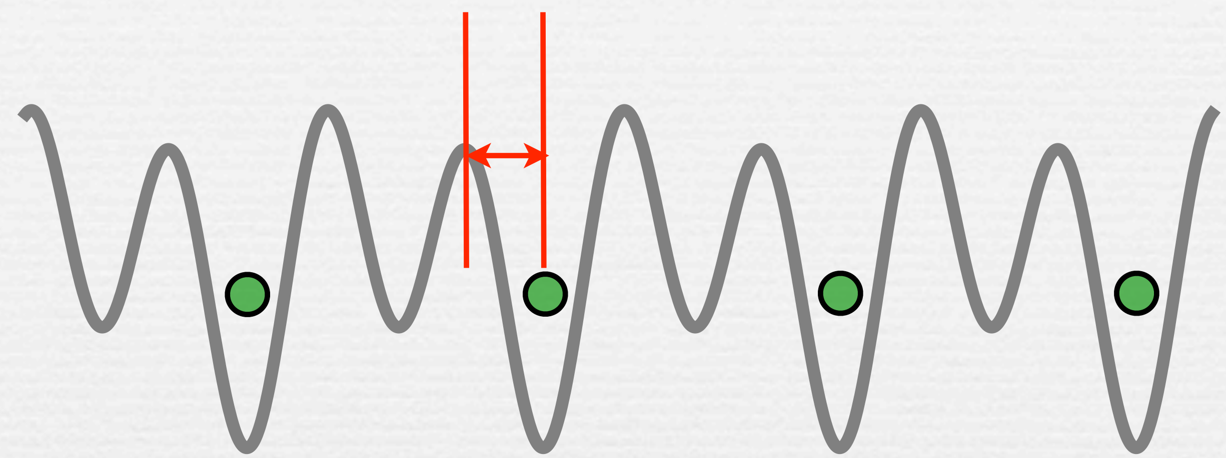


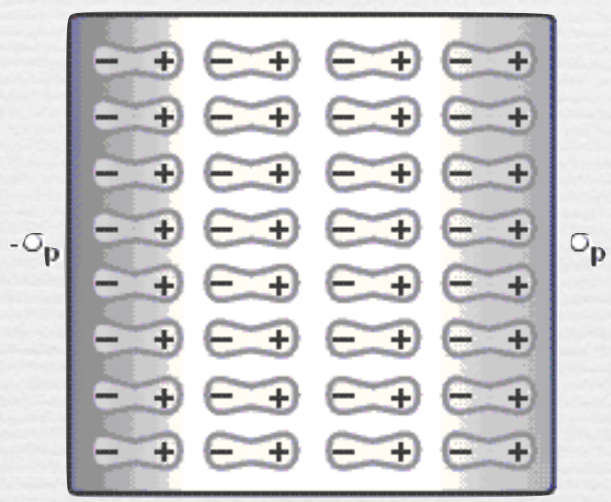
Polarization and Berry phase

Resta, King-Smith, Vanderbilt, ...

$$\text{“}x = i\partial_{k_x}\text{”}$$

$$P(t) = \int_0^{2\pi} \frac{\mathrm{d}k_x}{2\pi i} \langle u(k_x,t)|\partial_{k_x}|u(k_x,t)\rangle$$





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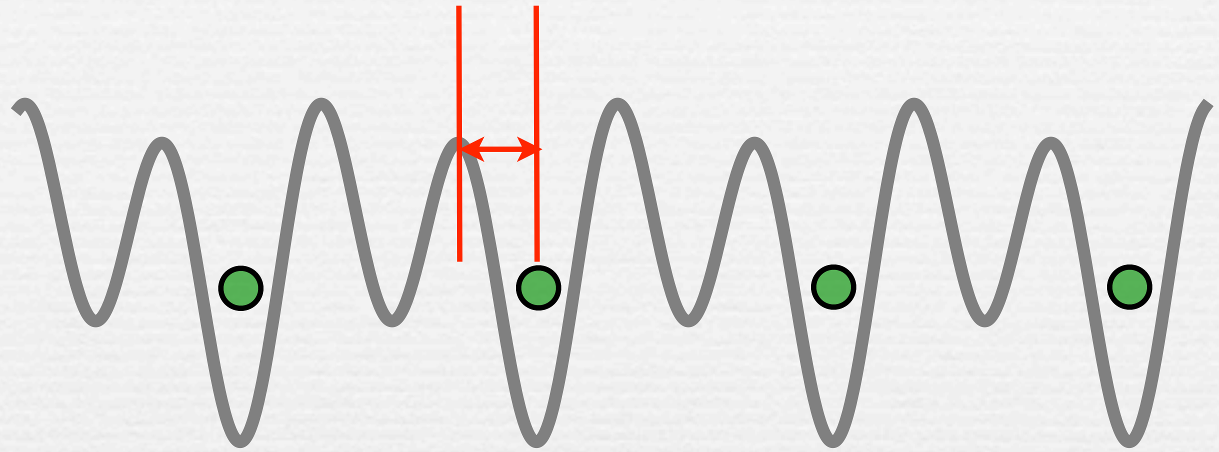
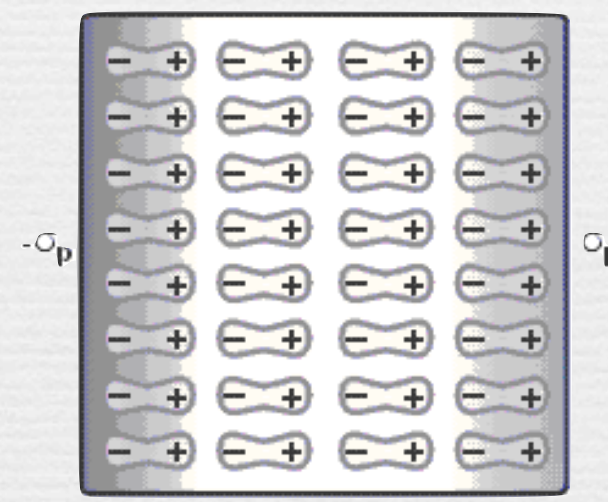


Change of Polarization



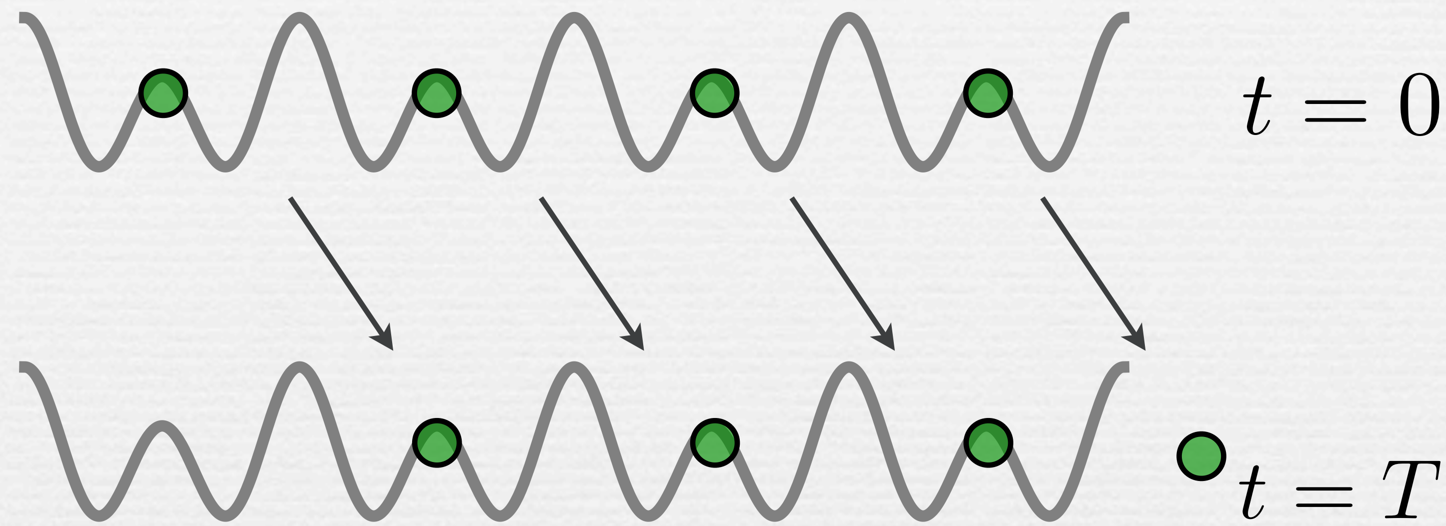
$$\Delta P = \int_0^T \mathrm{d}P$$

Berry Curvature

$$= \frac{1}{2\pi i}\int_0^T \int_0^{2\pi} \mathrm{d}t\,\mathrm{d}k_x \boxed{\left(\langle \partial_t u|\partial_{k_x} u\rangle - h.c.\right)}$$

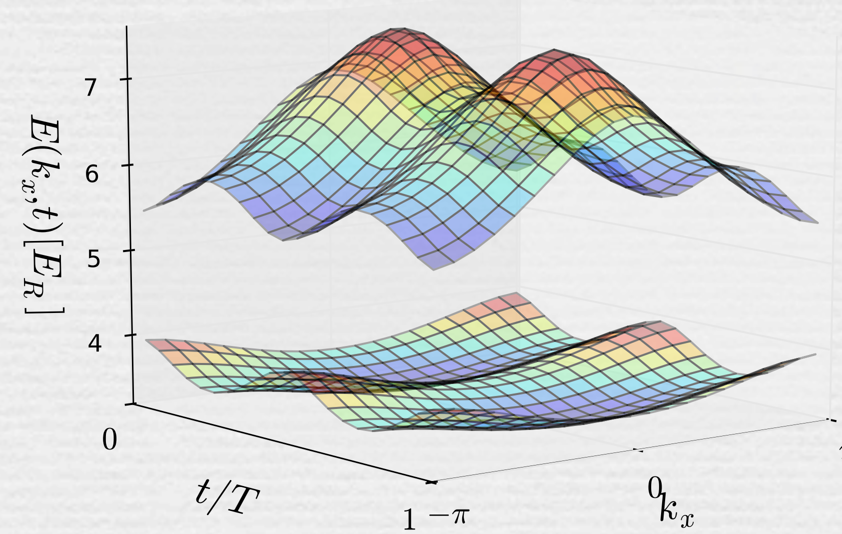
$=$ Chern number

Connection to IQHE

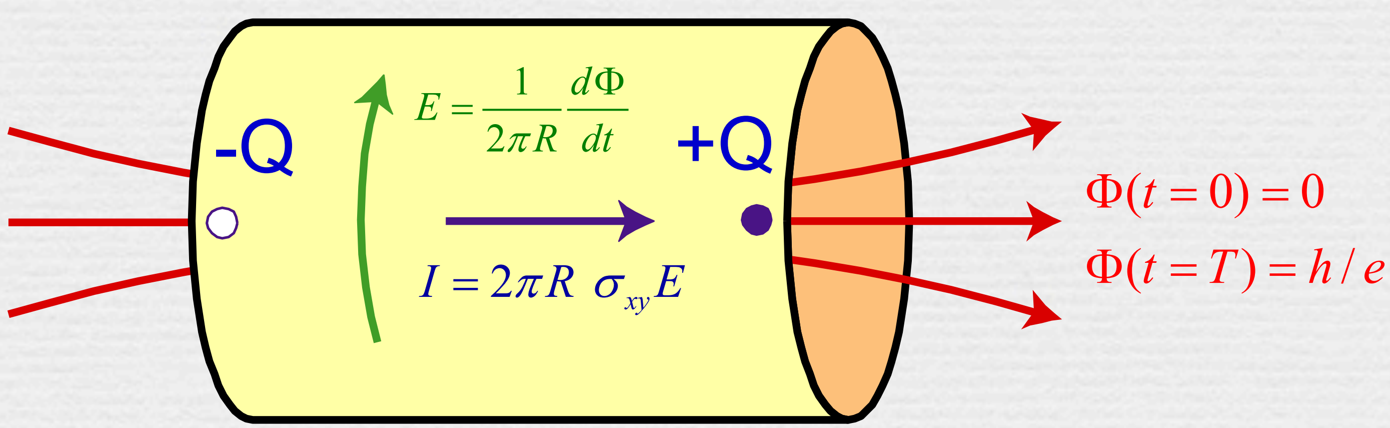


$$\boxed{H(k_x, t) = H(k_x, t + T)}$$

$V_1 = 4E_R$ $V_2 = 4E_R$



Adiabatically thread a quantum of magnetic flux through cylinder.



Laughlin, 1981

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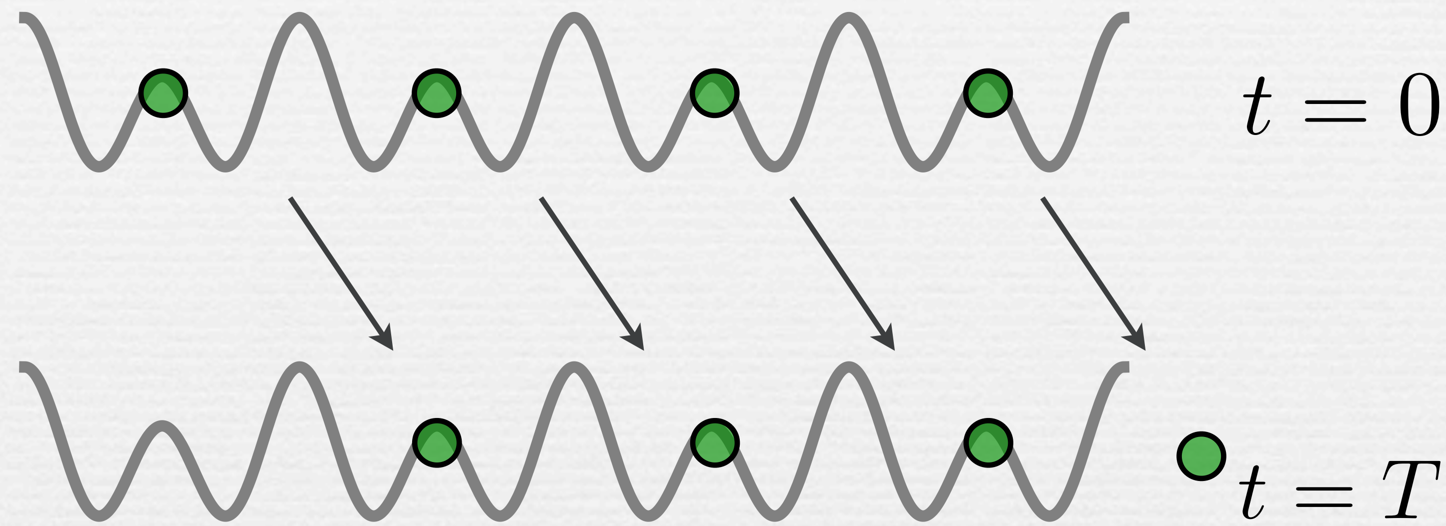

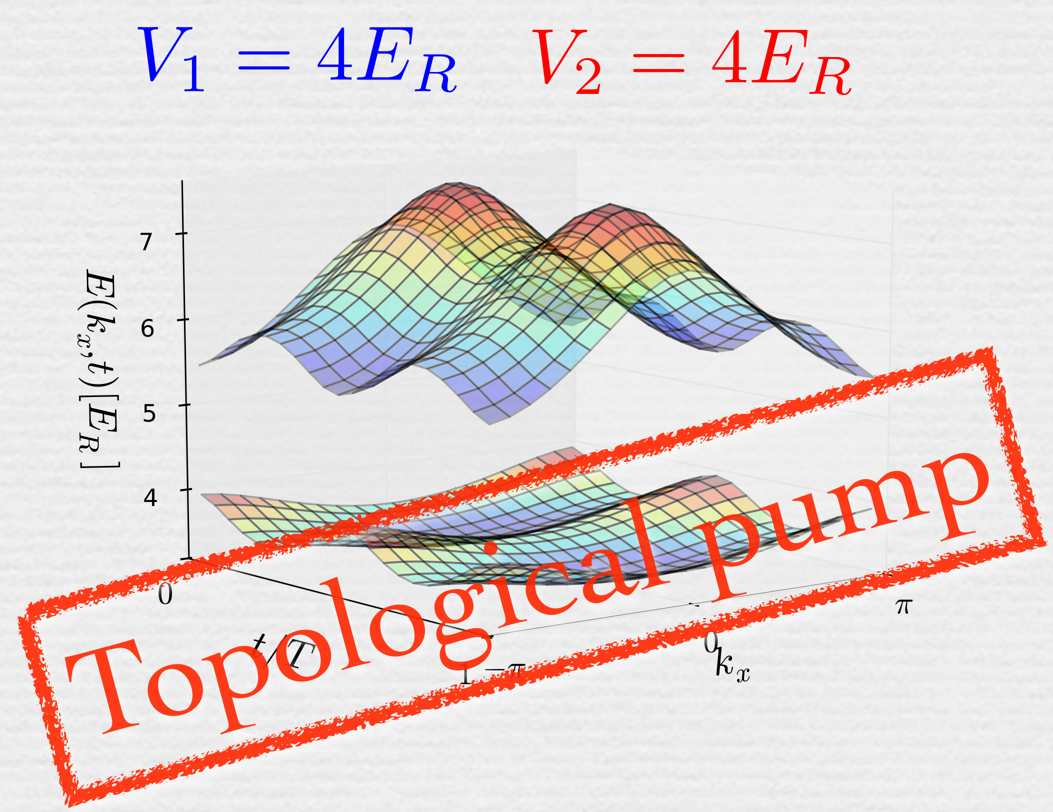

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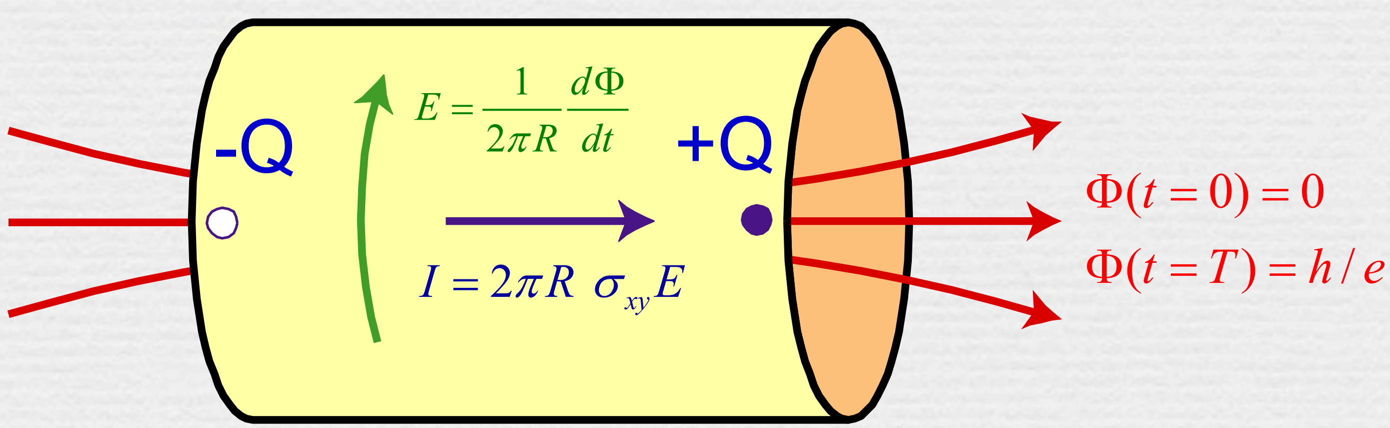

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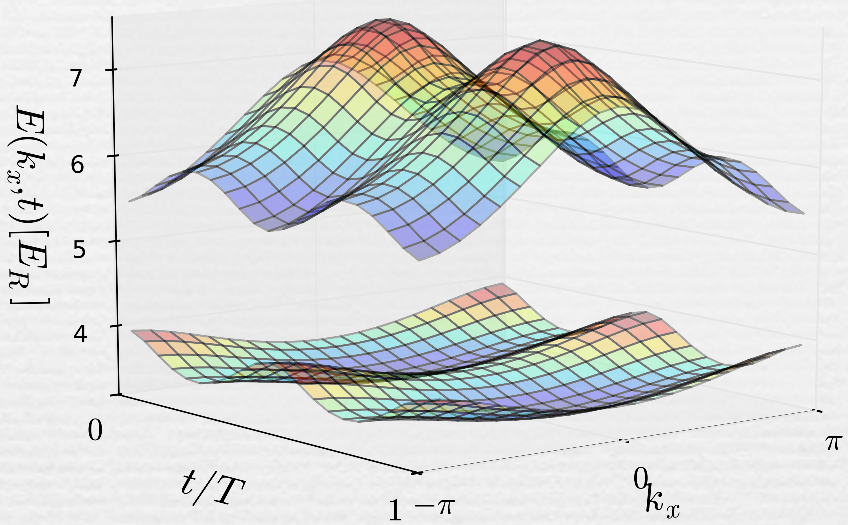
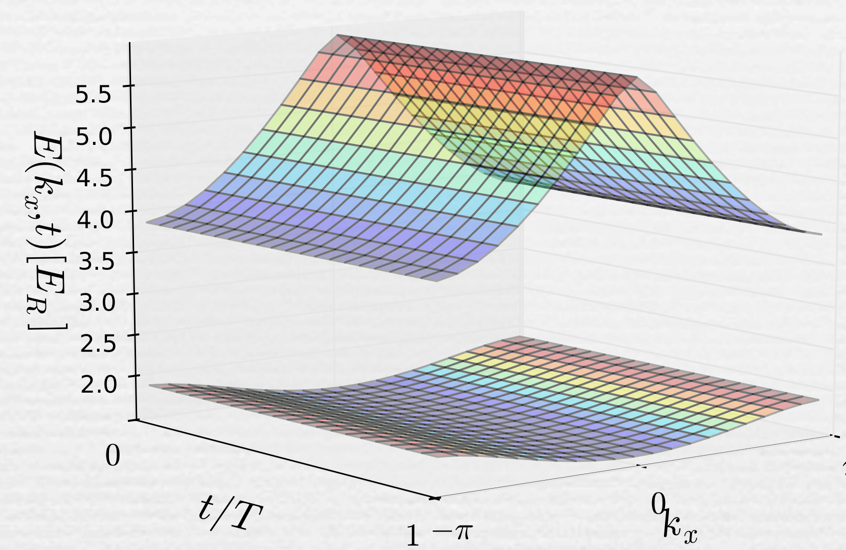
Adiabatic connection

$V_1 = 4E_R \quad V_2 = 4E_R$



$$V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

$V_1 = 0 \quad V_2 = 4E_R$



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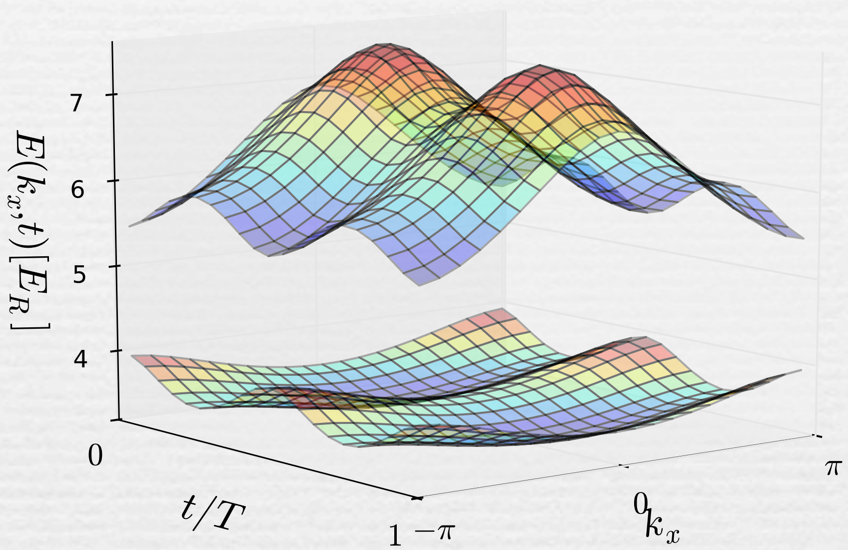

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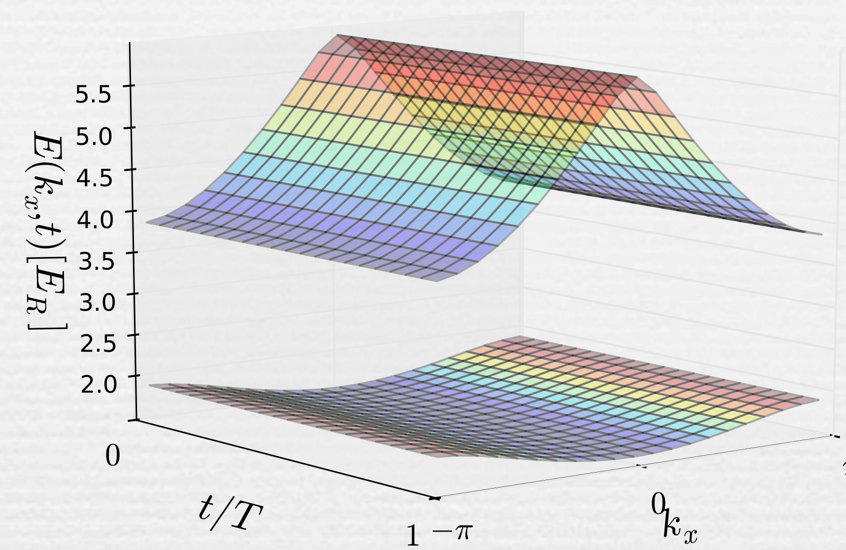

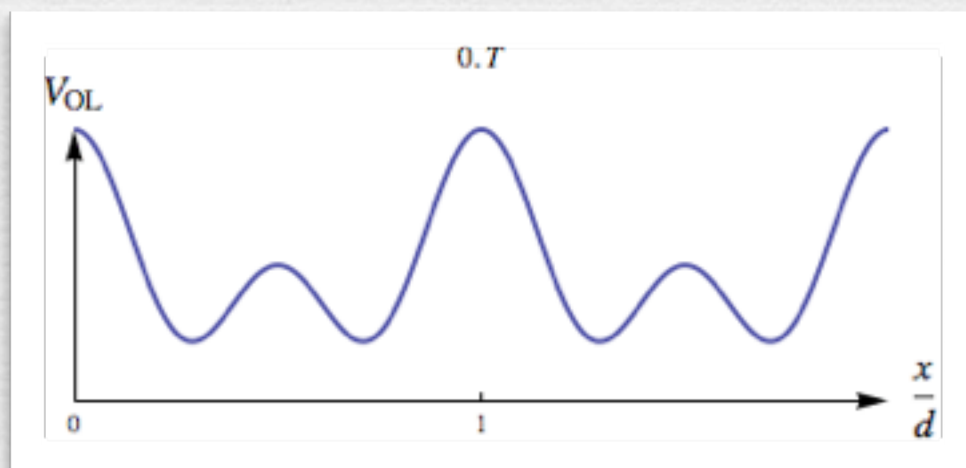

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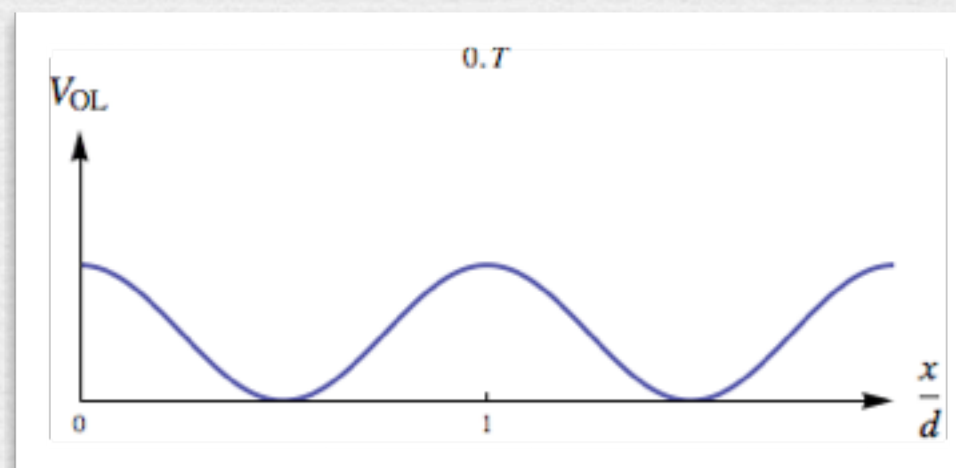
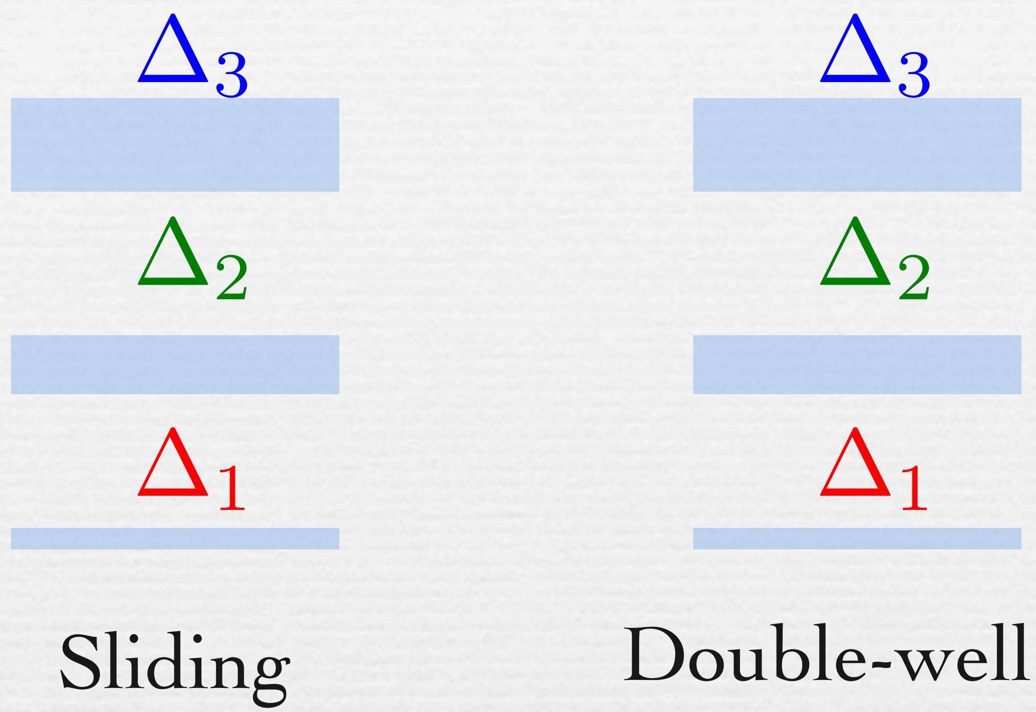
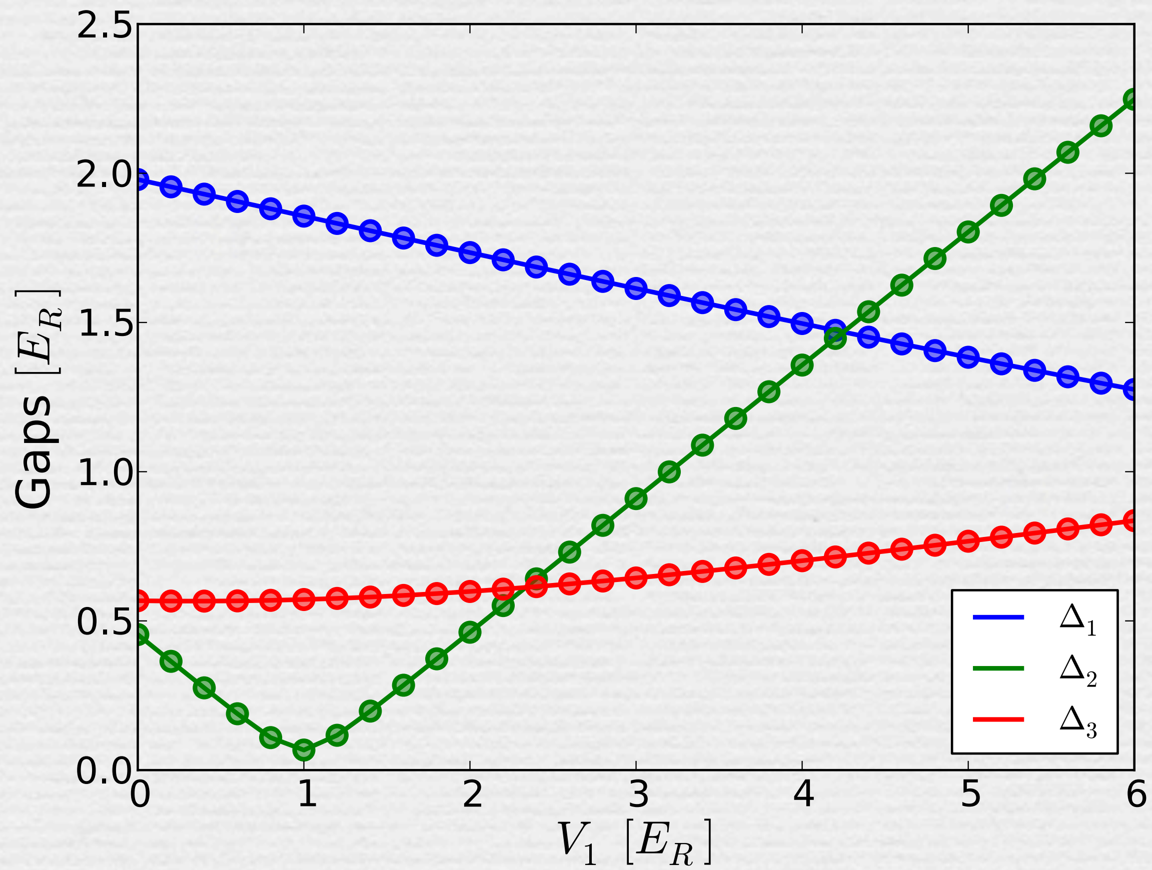


Higher bands

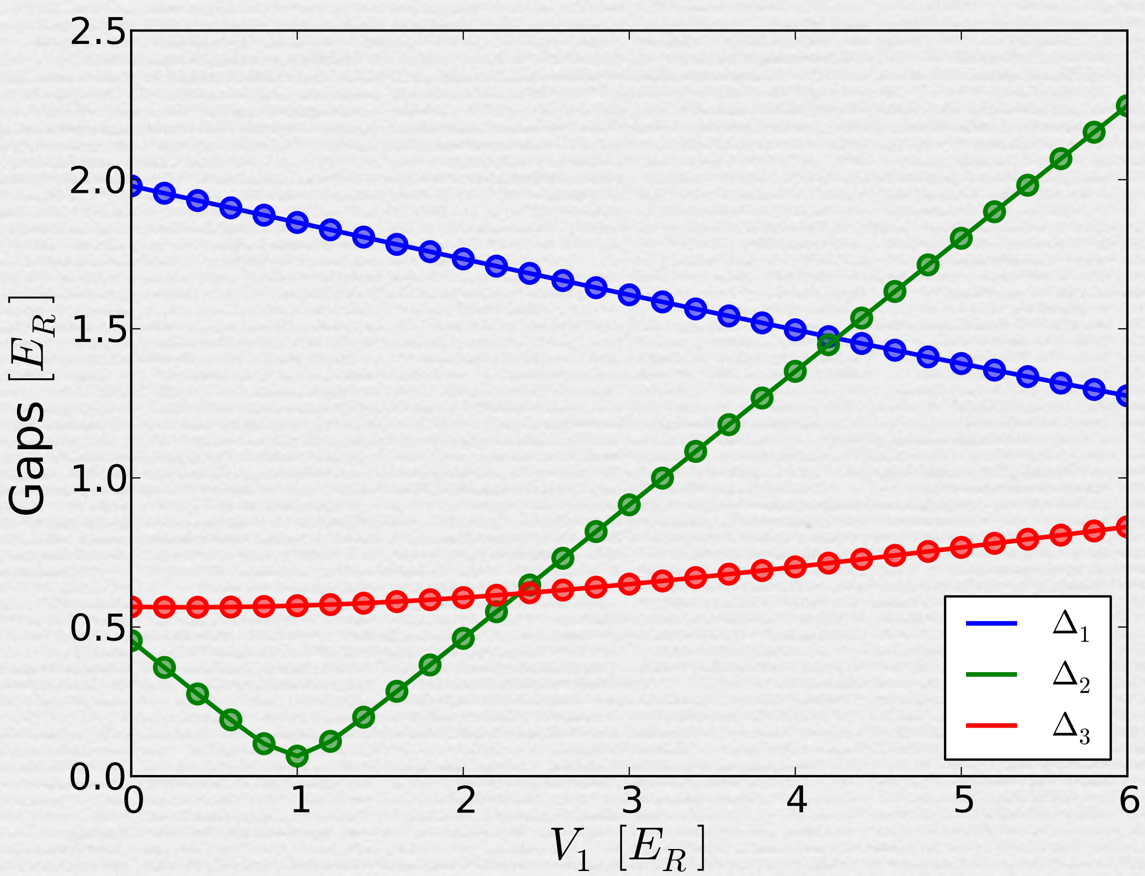




Higher bands

Sliding

Double-well



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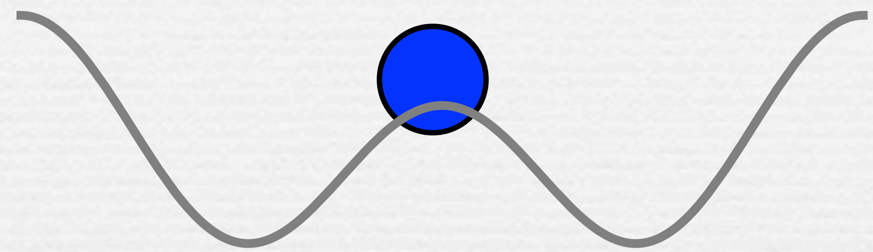

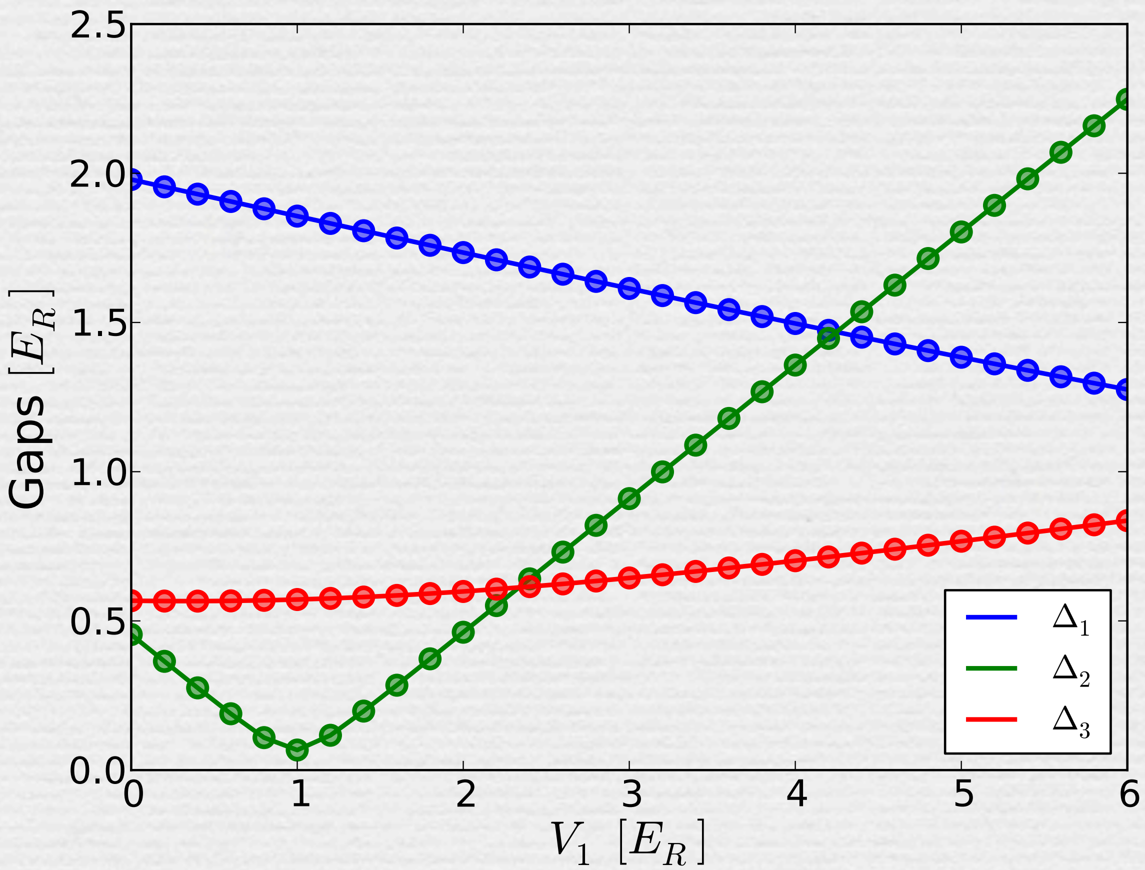

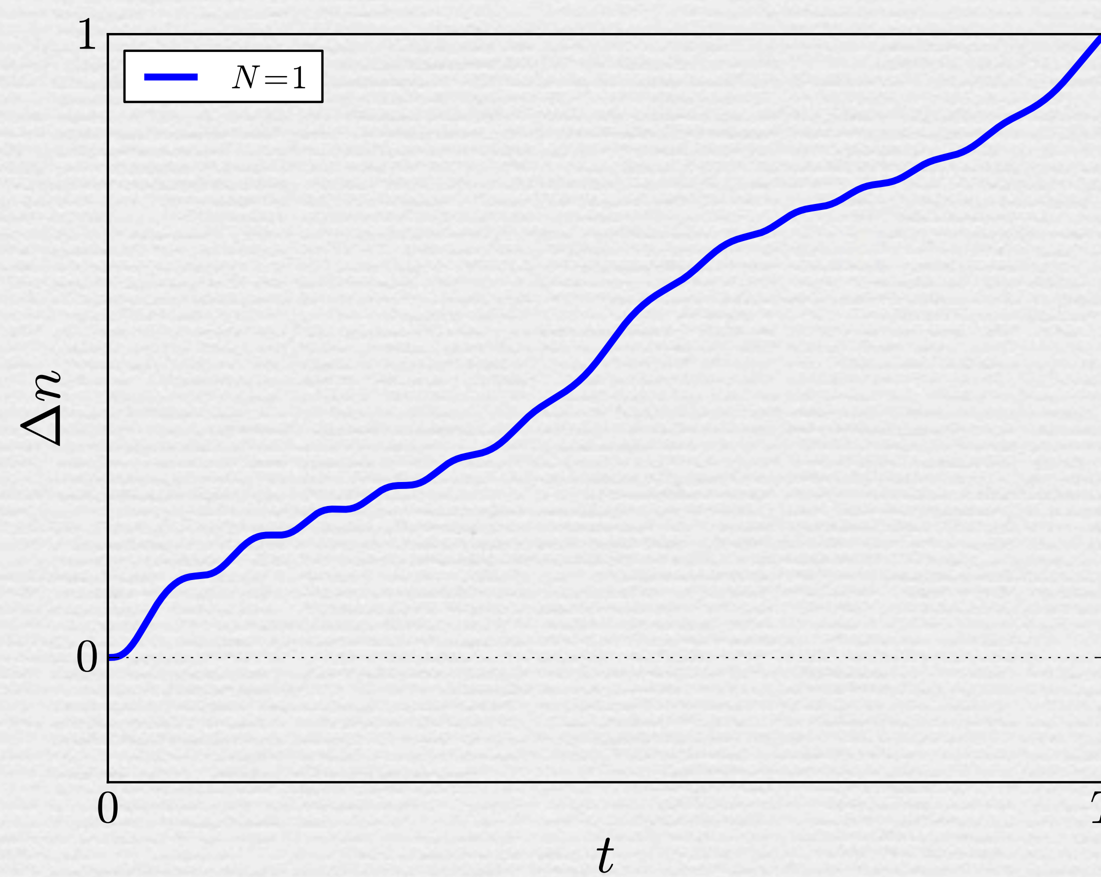

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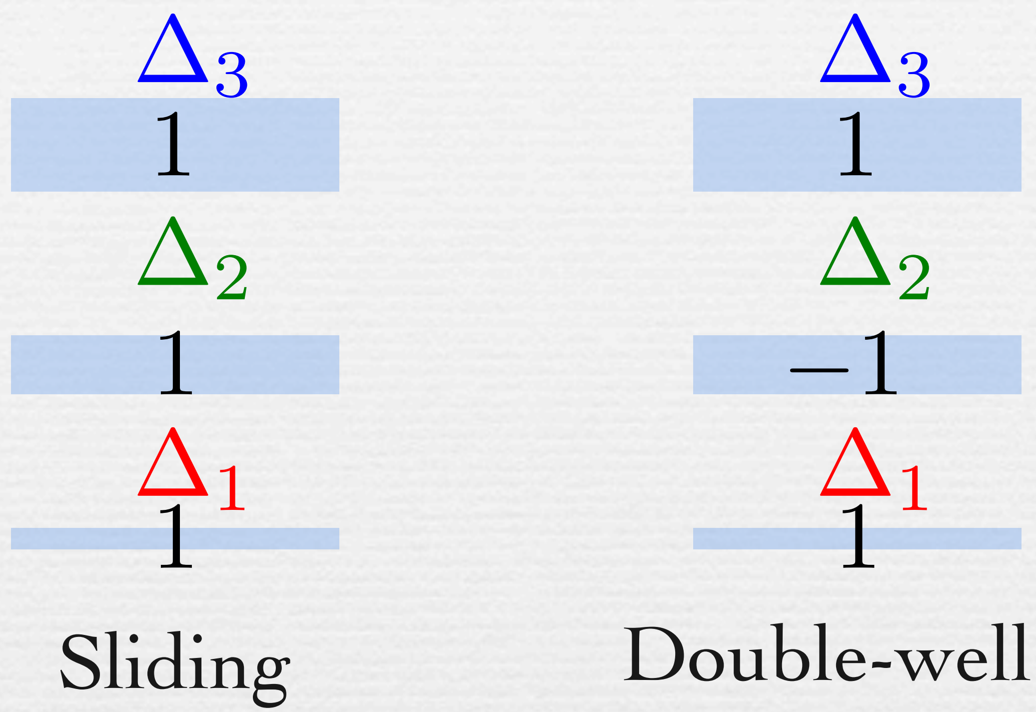

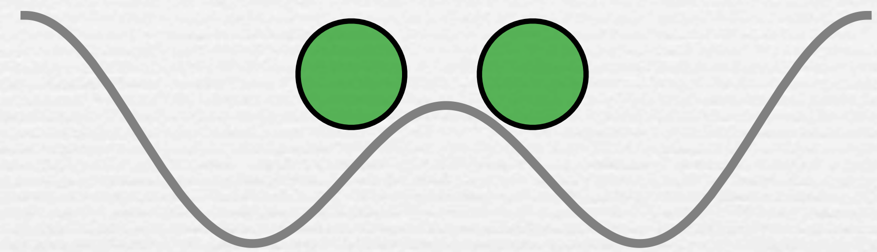

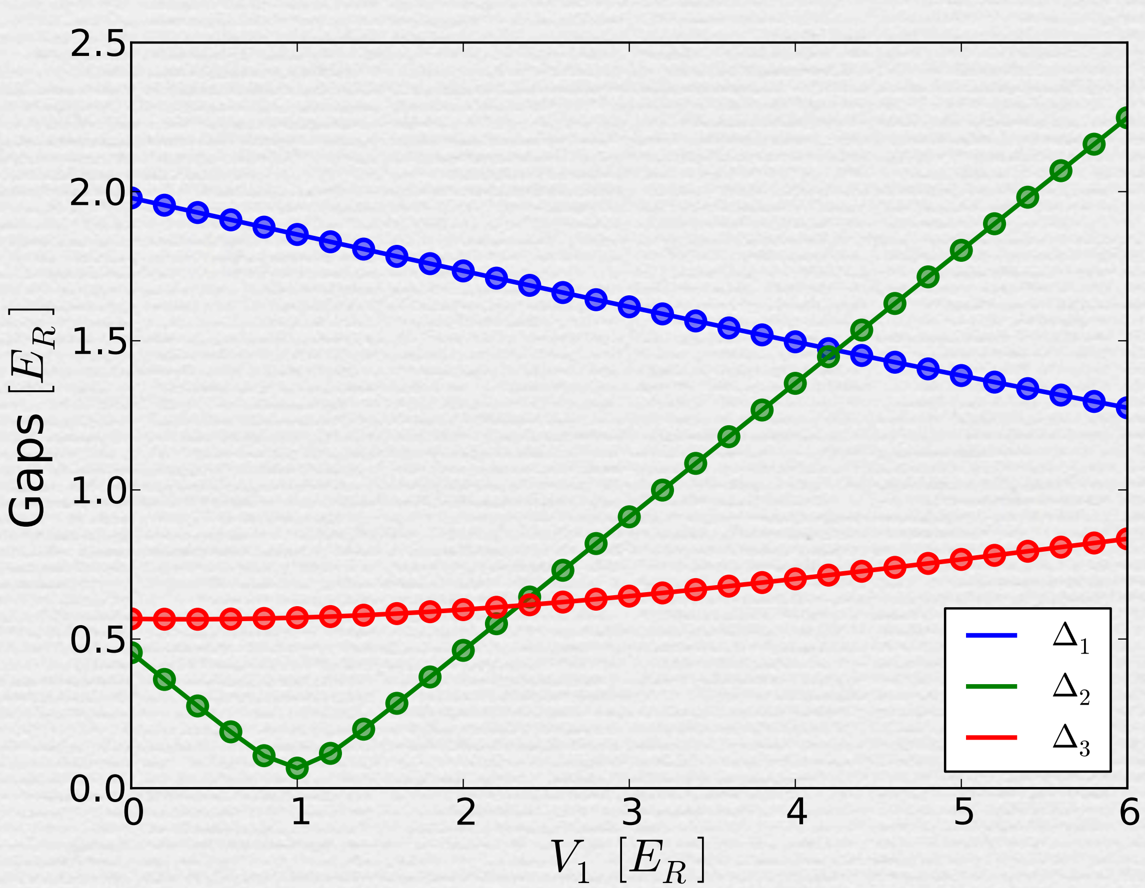

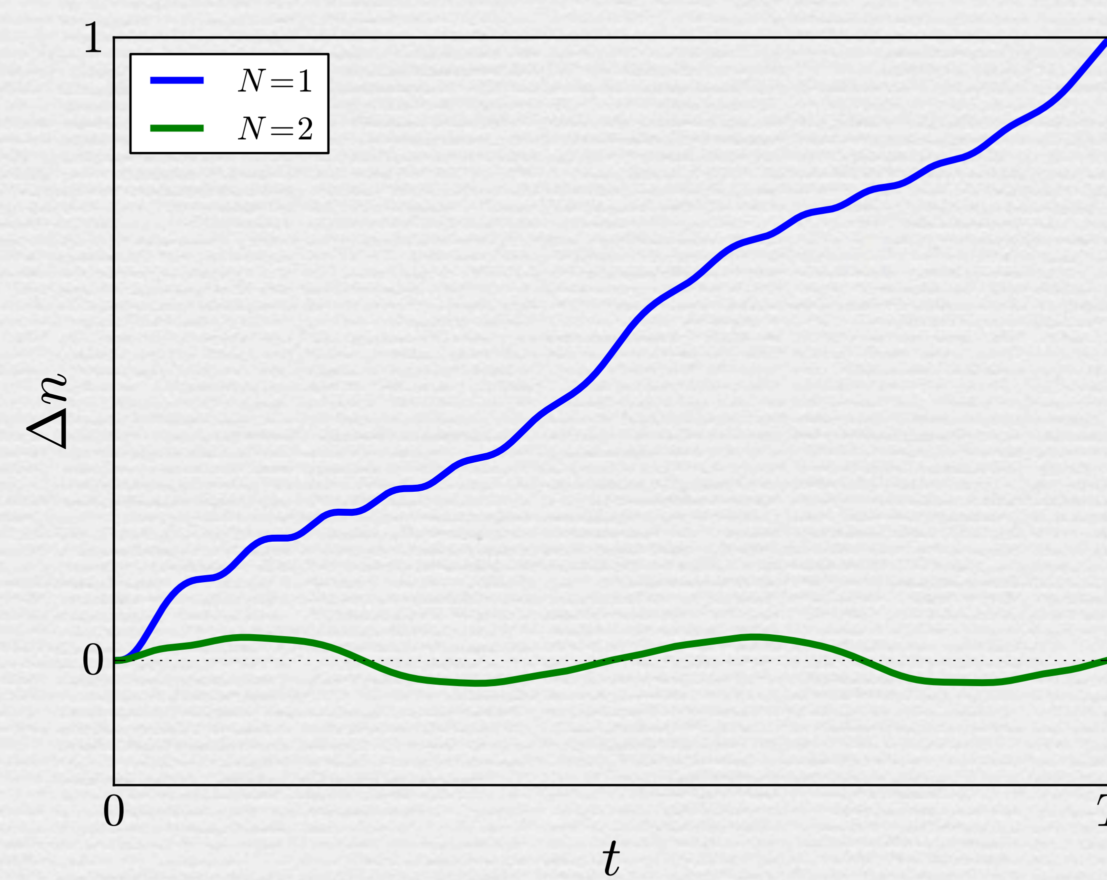

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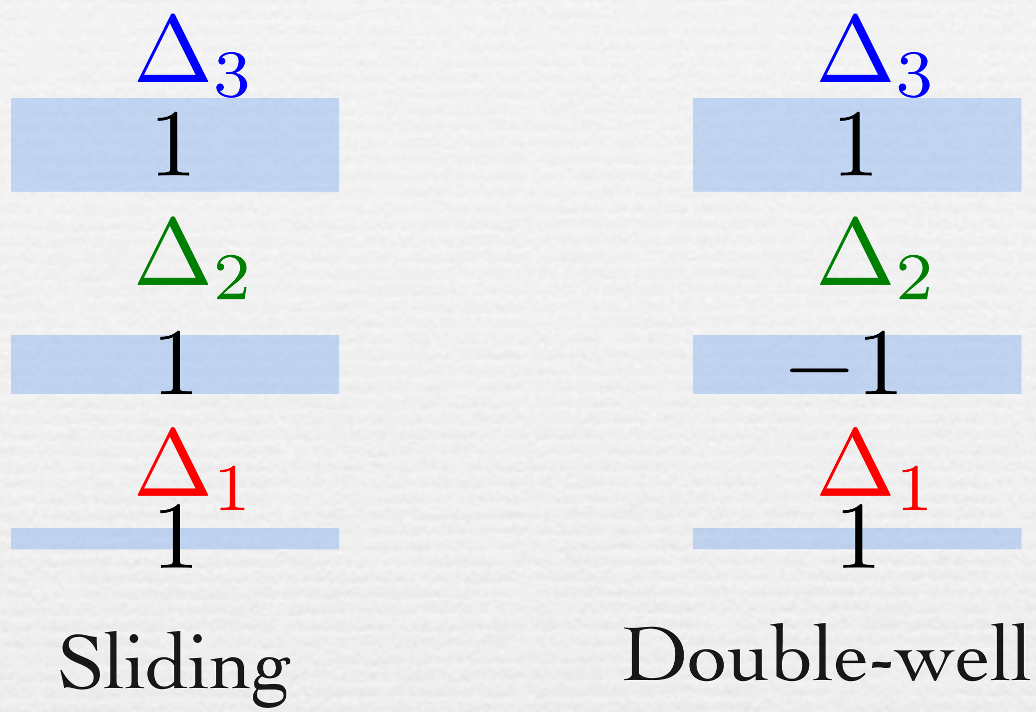
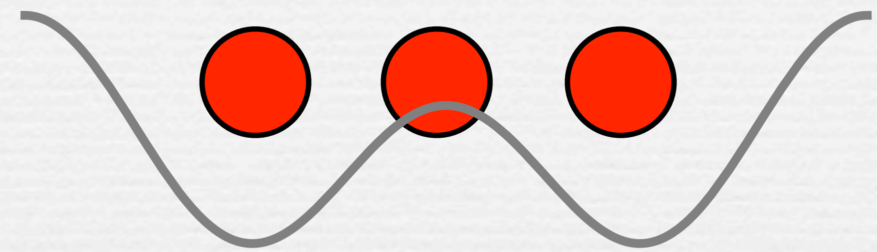
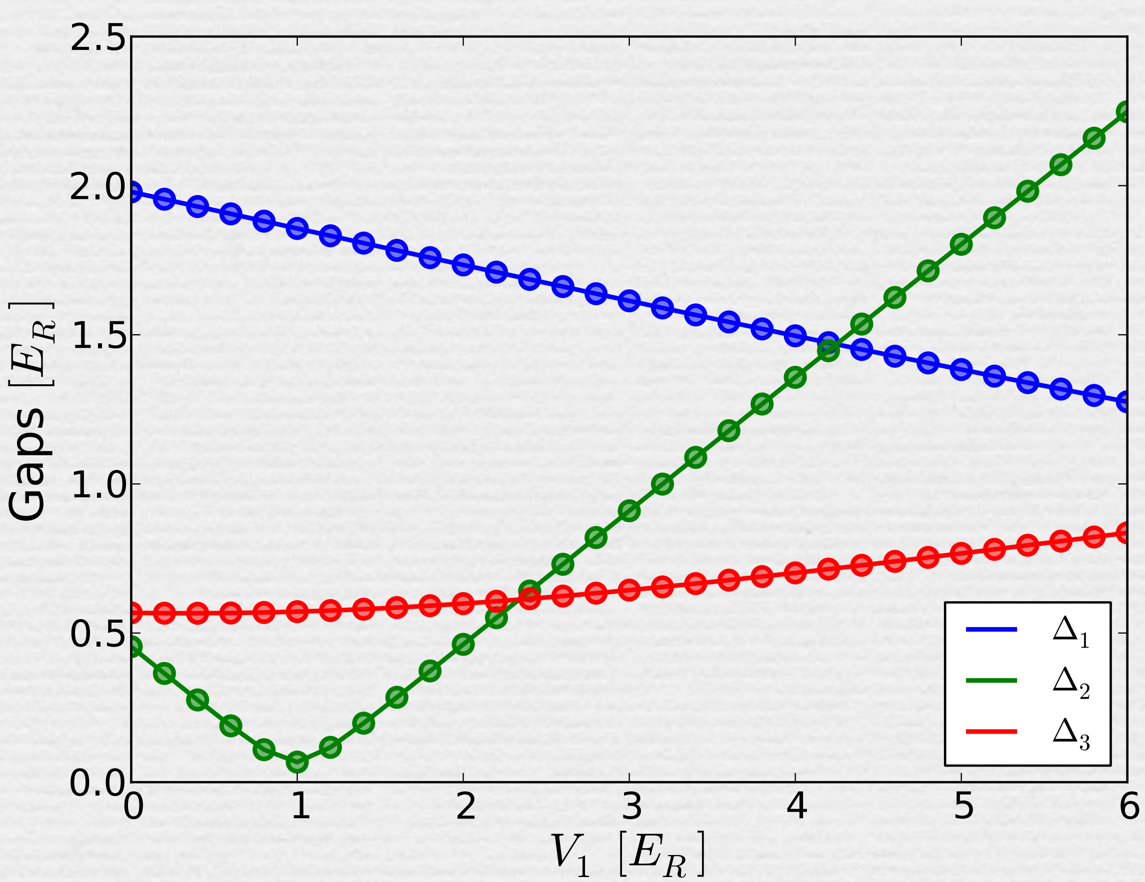
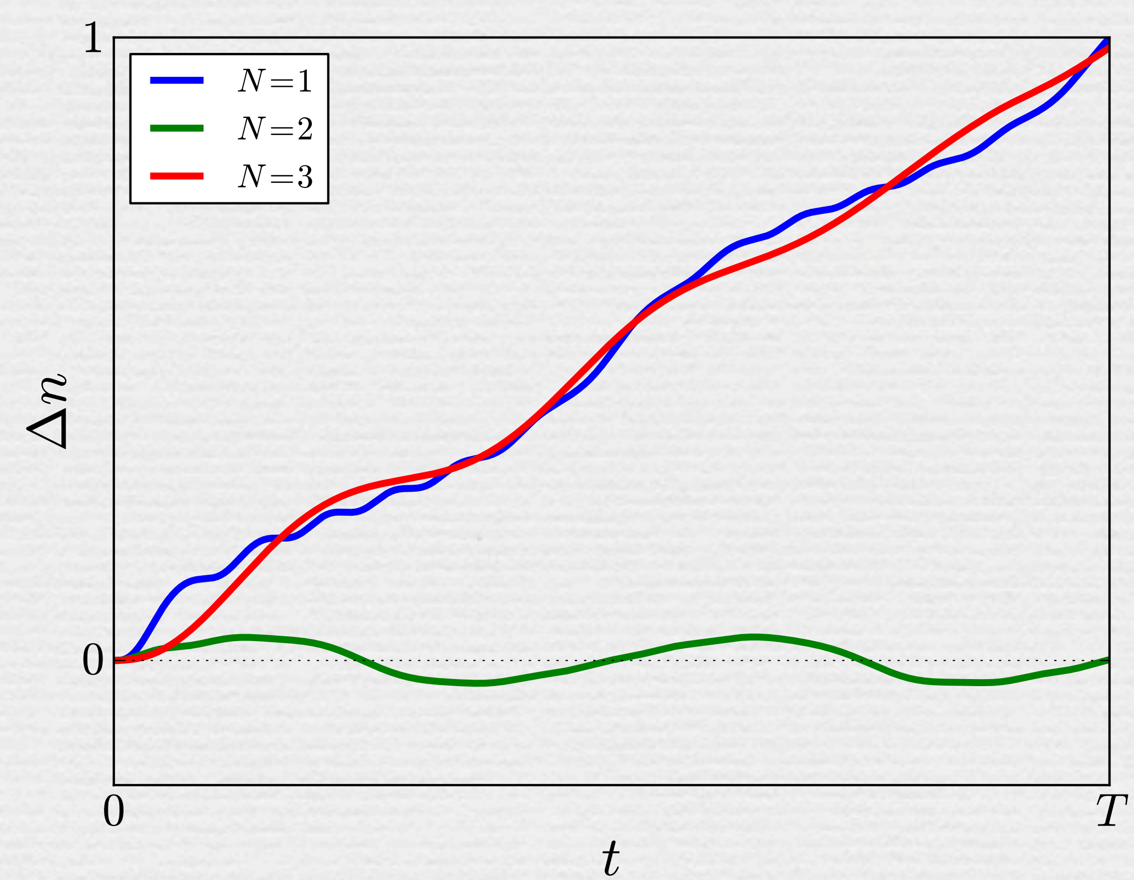


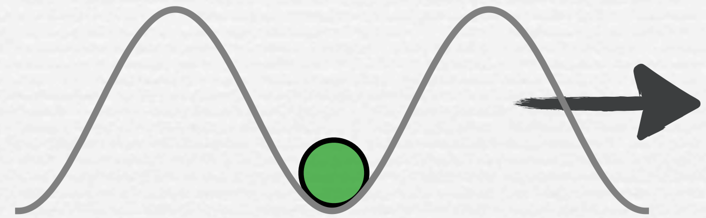






Classical dynamics

$$\boxed{m\ddot{x} = -\frac{\partial V_{\mathrm{OL}}(x,t)}{\partial x}}$$



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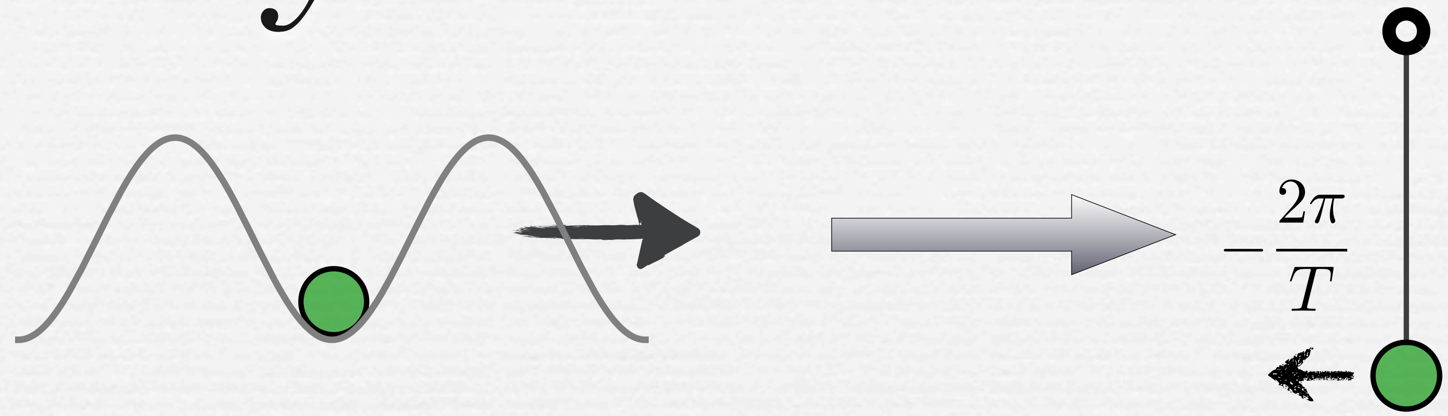

- Without V_1 term, pumping maps to a simple pendulum

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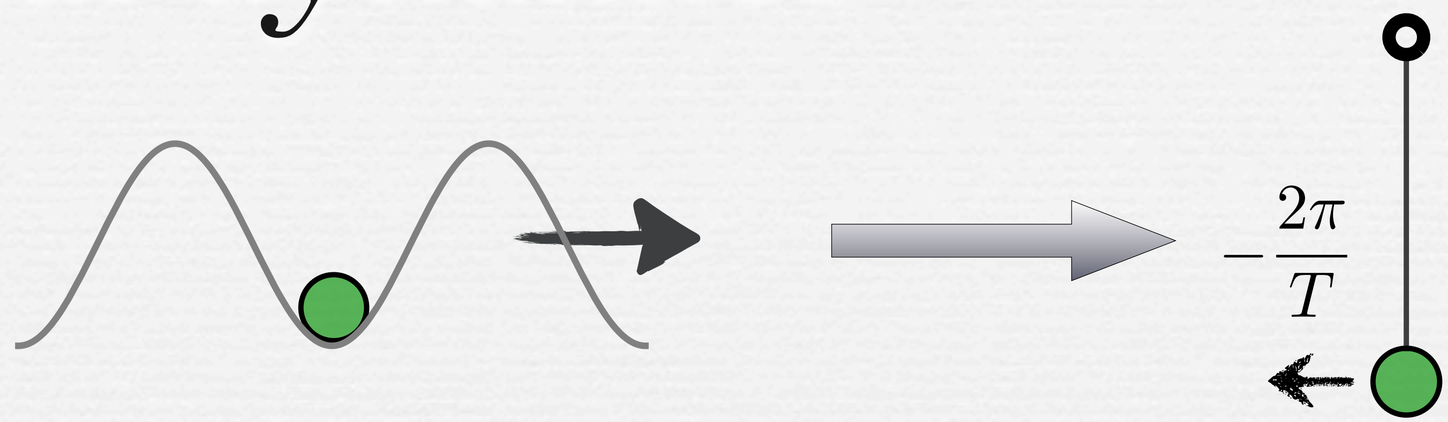

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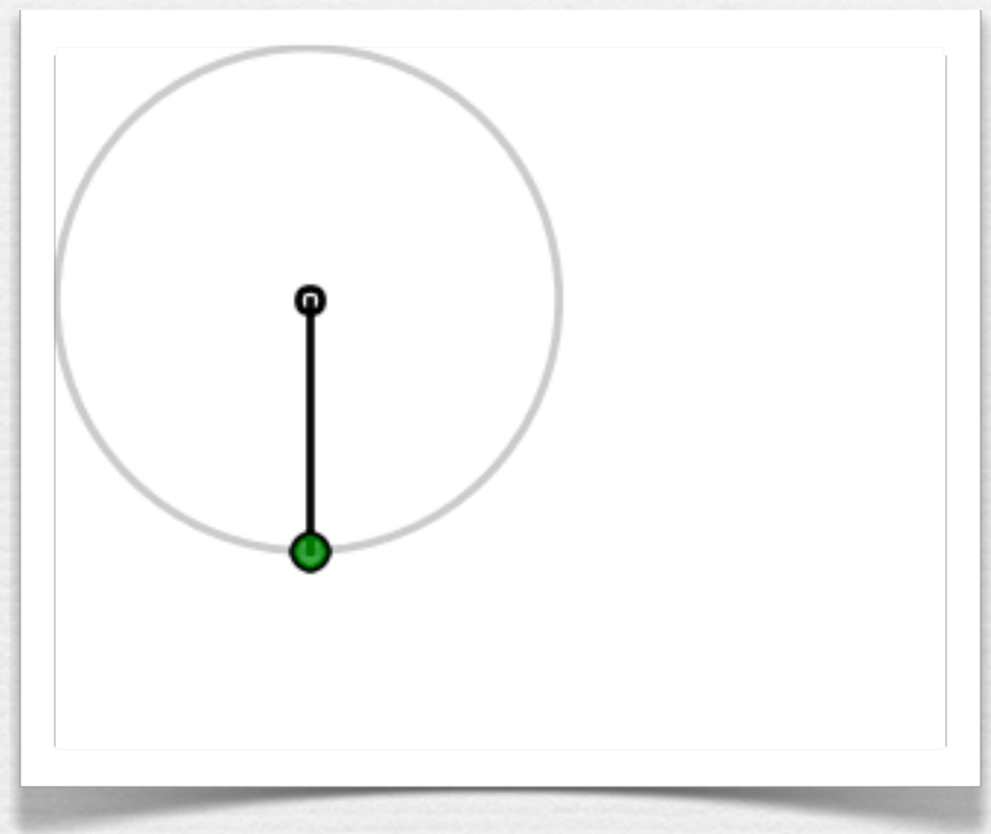

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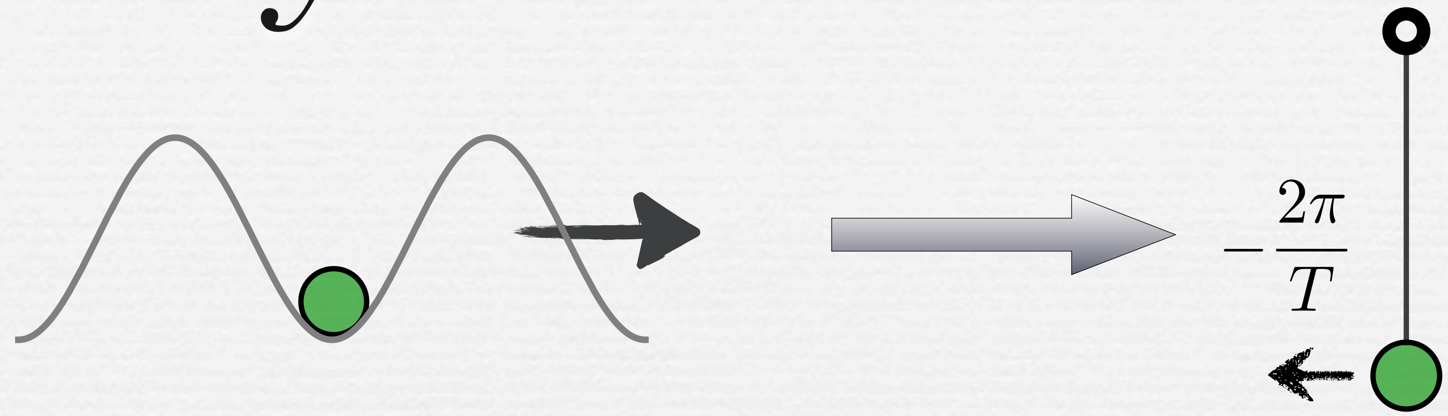

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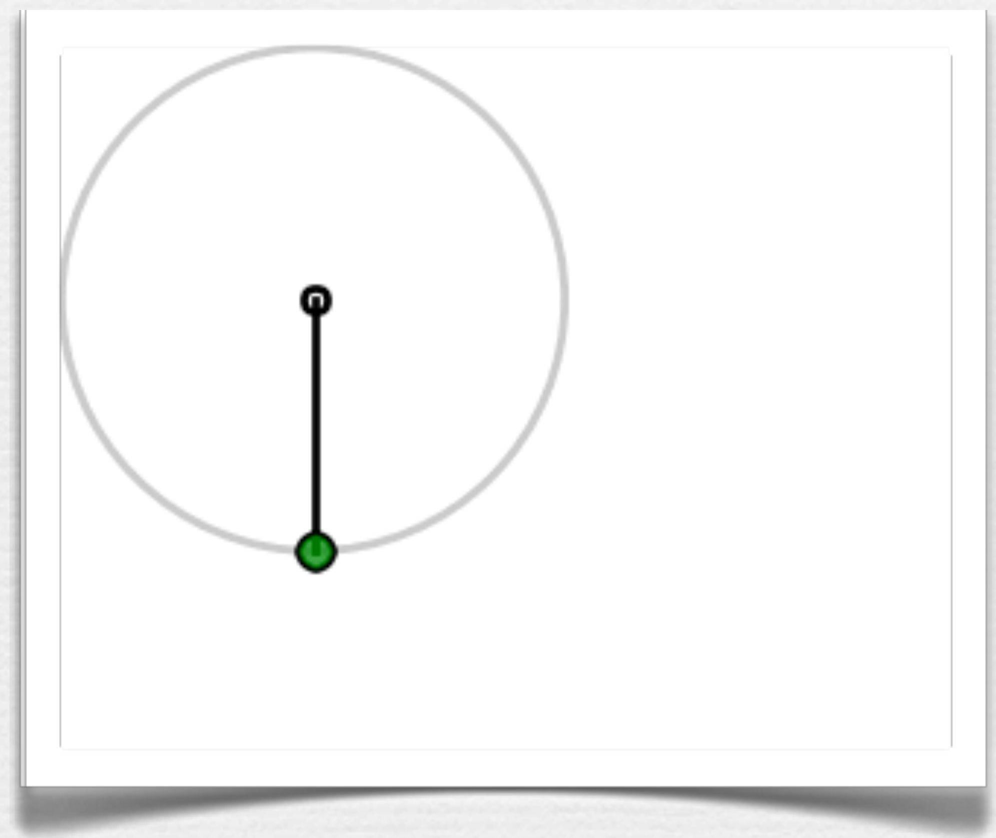

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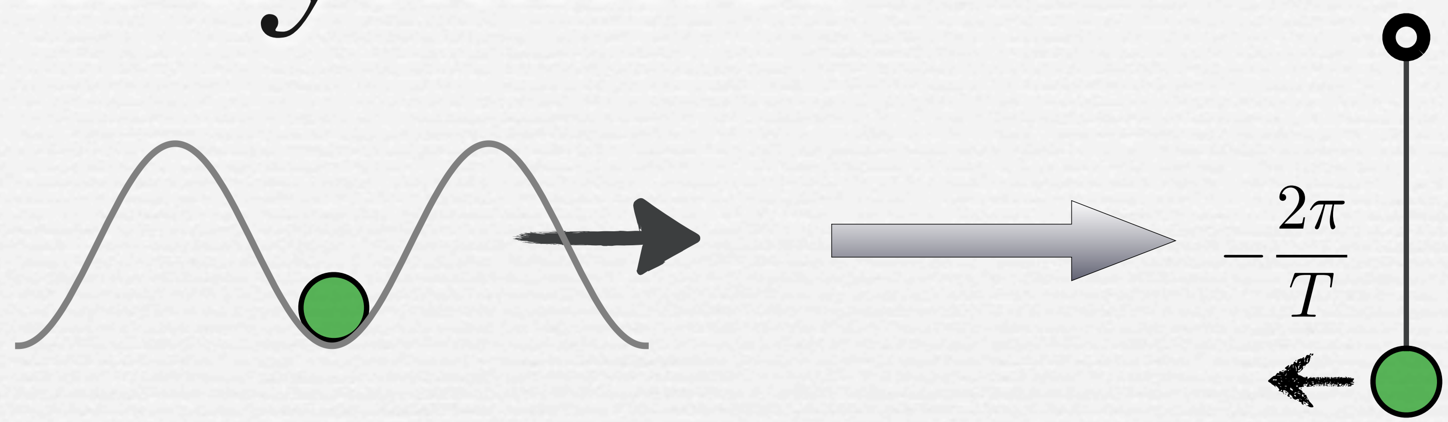

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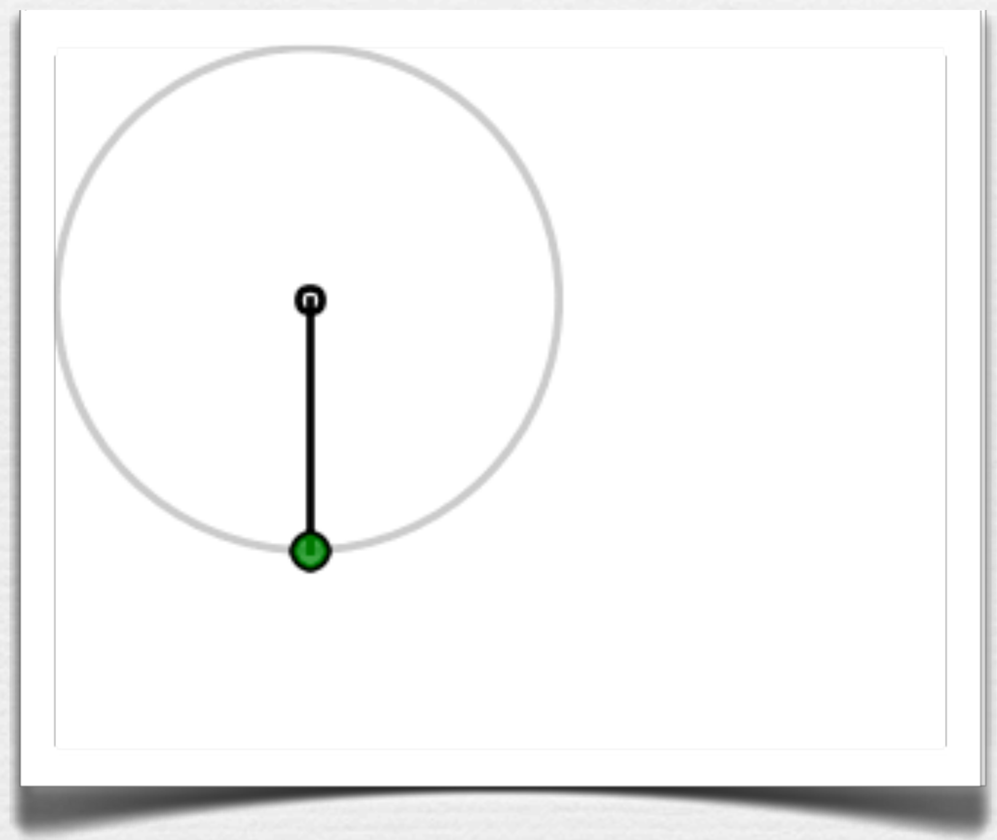

- V_1 term: Driven pendulum shows chaotic behavior.

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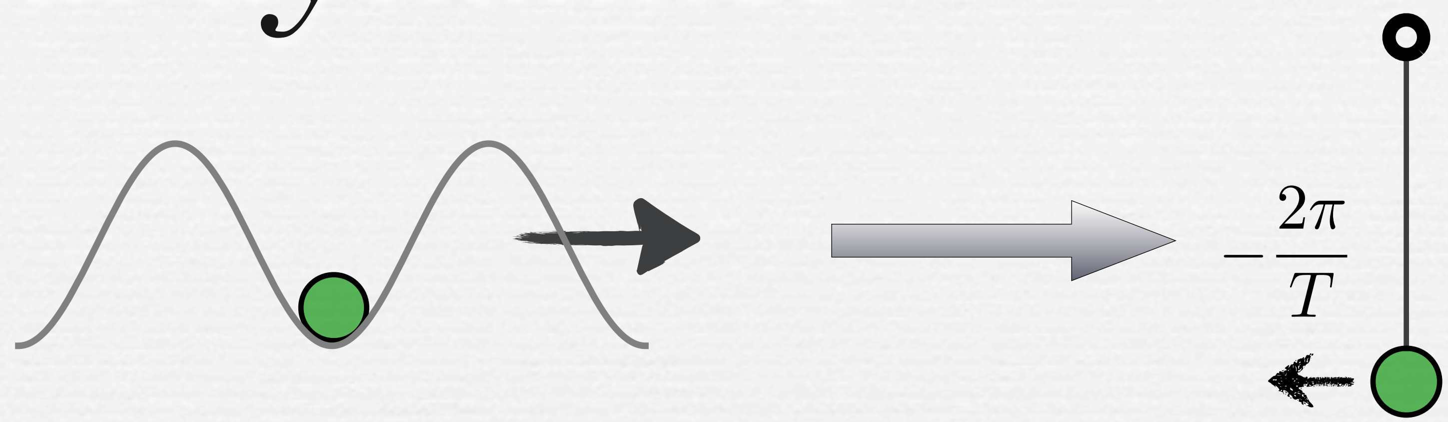

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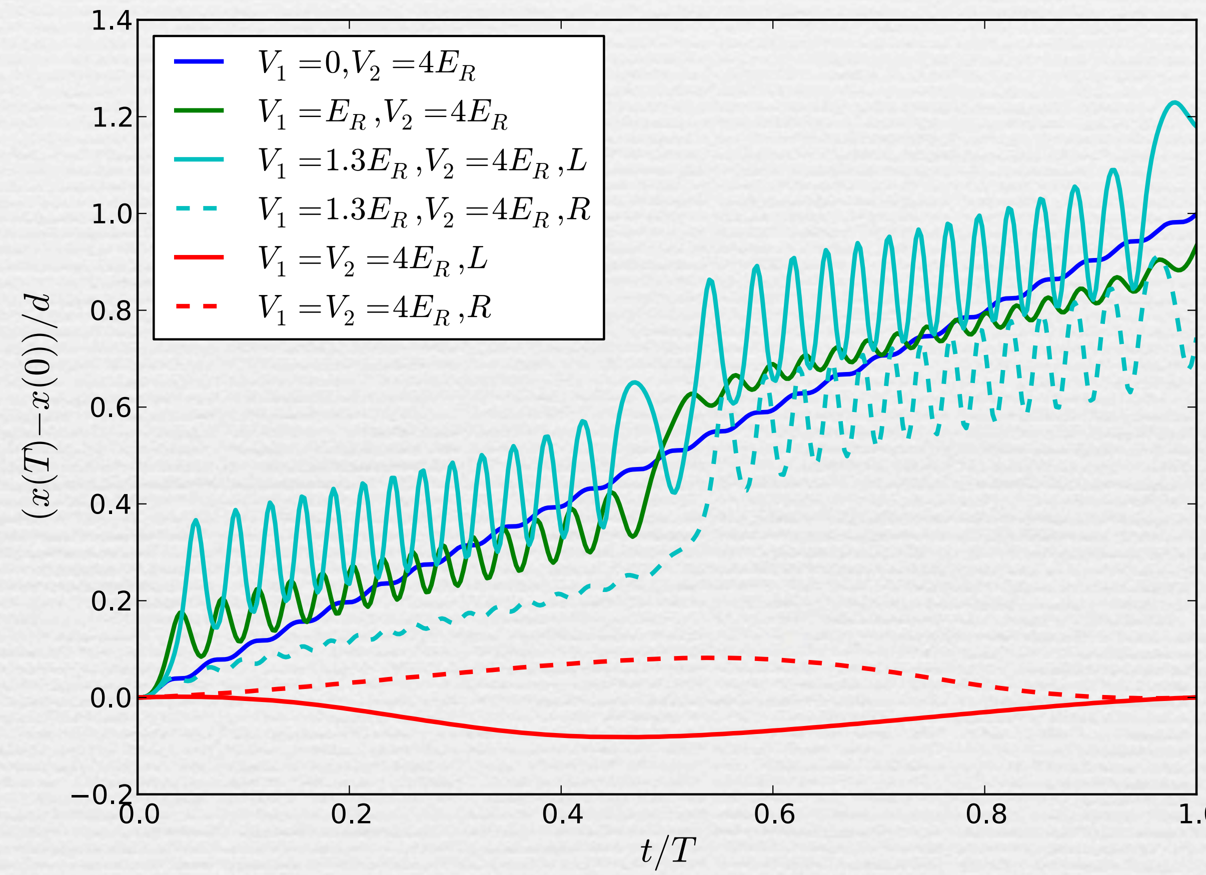

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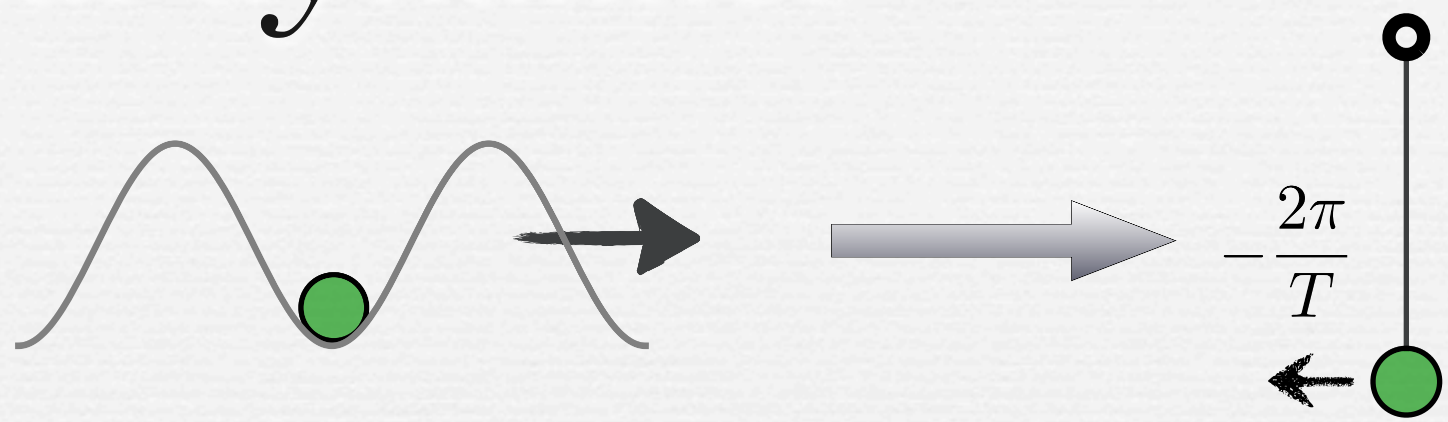

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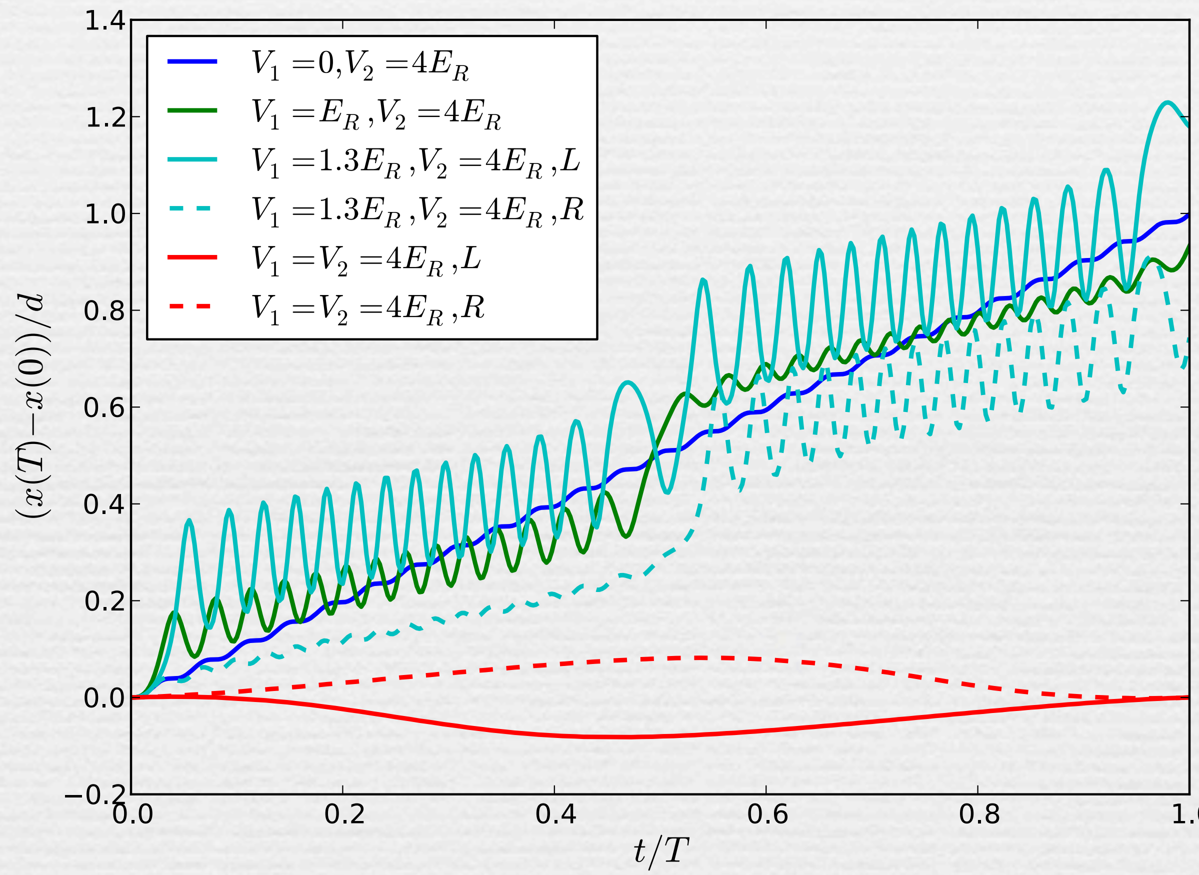


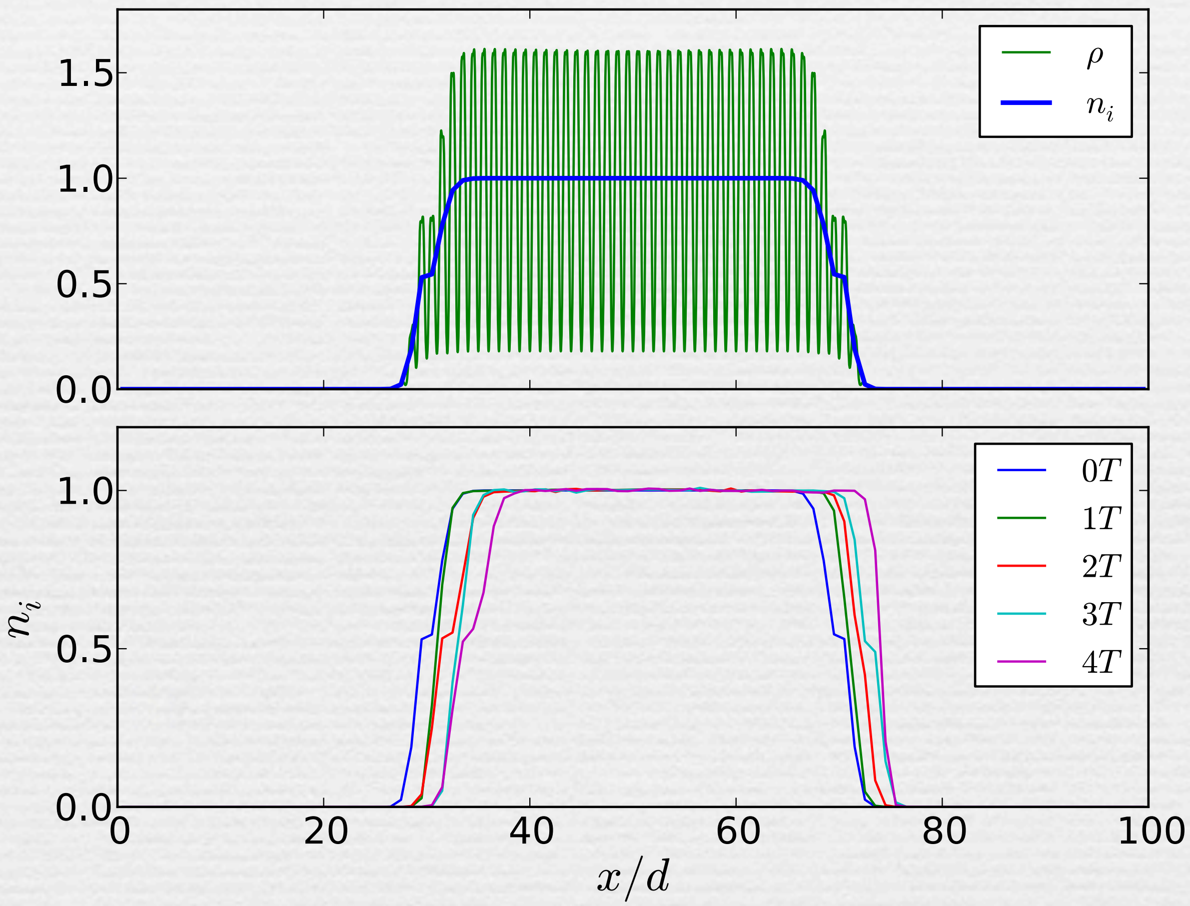
In general, classical pumped charge is not quantized

Practical issues

- Detection
- External trap
- Temperature effect
- Non-adiabatic effect

Trapping & Detection

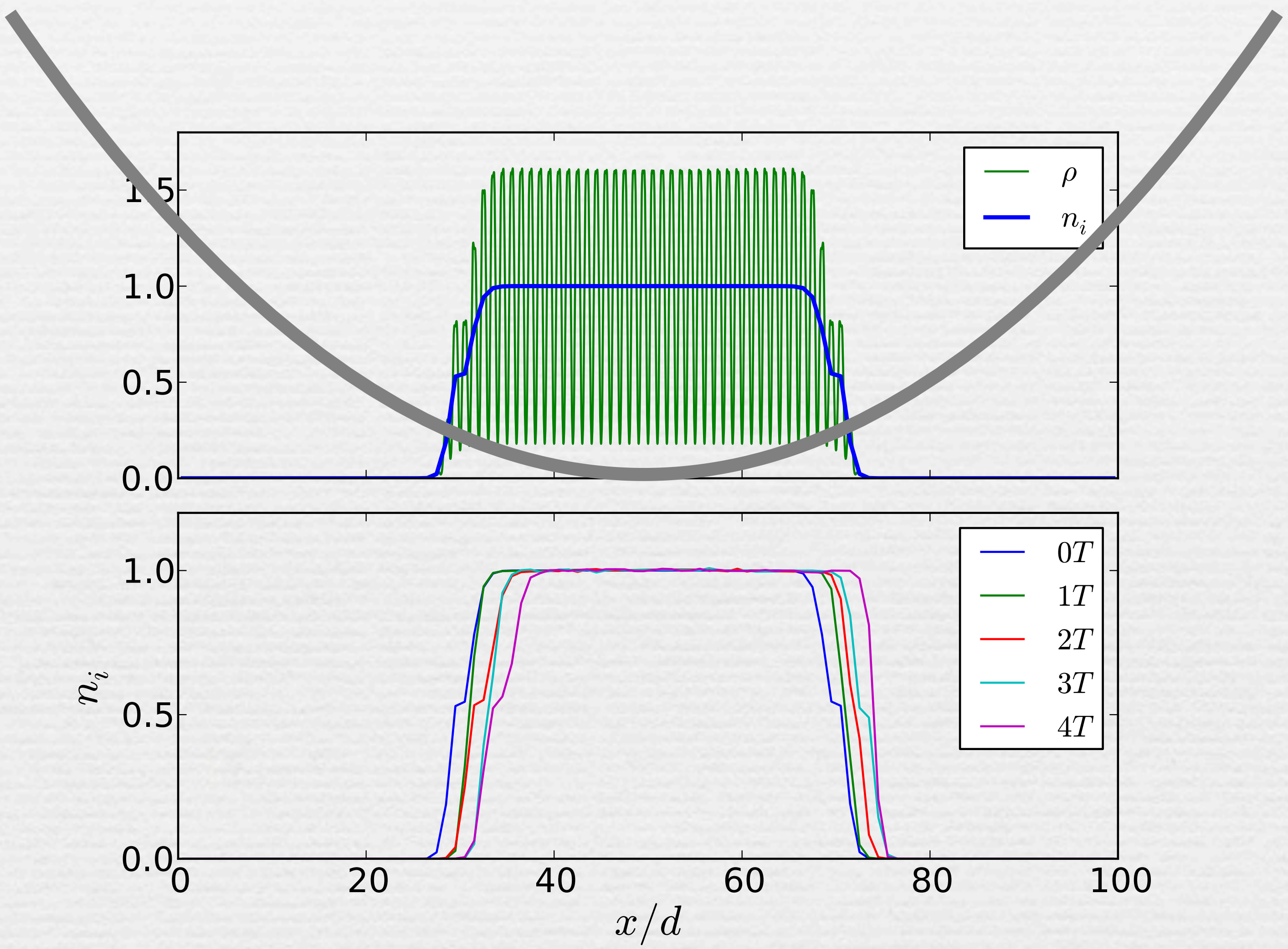
LW, Troyer and Dai, 1301.7435



$$\boxed{\langle x \rangle / d = \Delta n}$$

Trapping & Detection

LW, Troyer and Dai, 1301.7435



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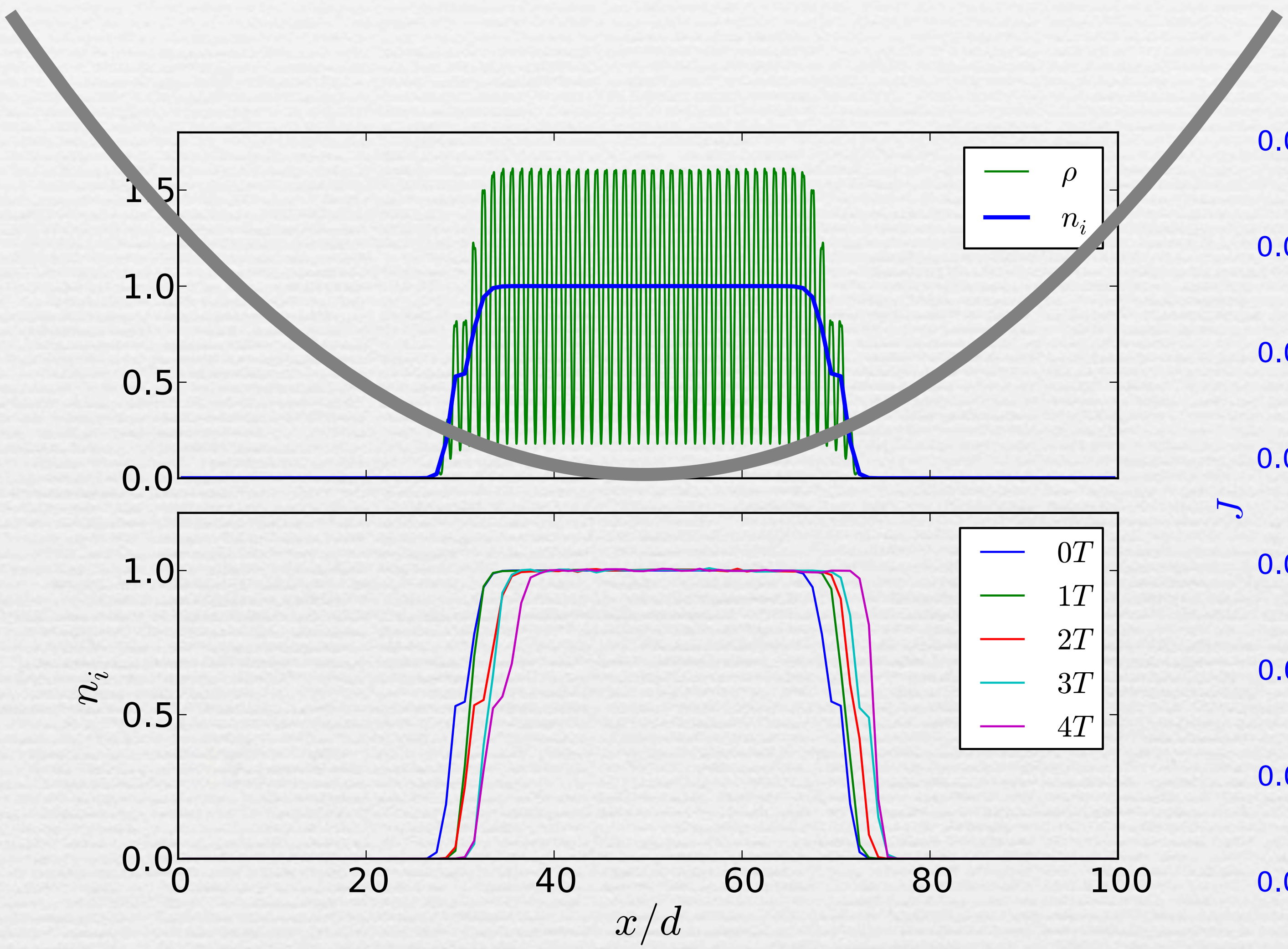

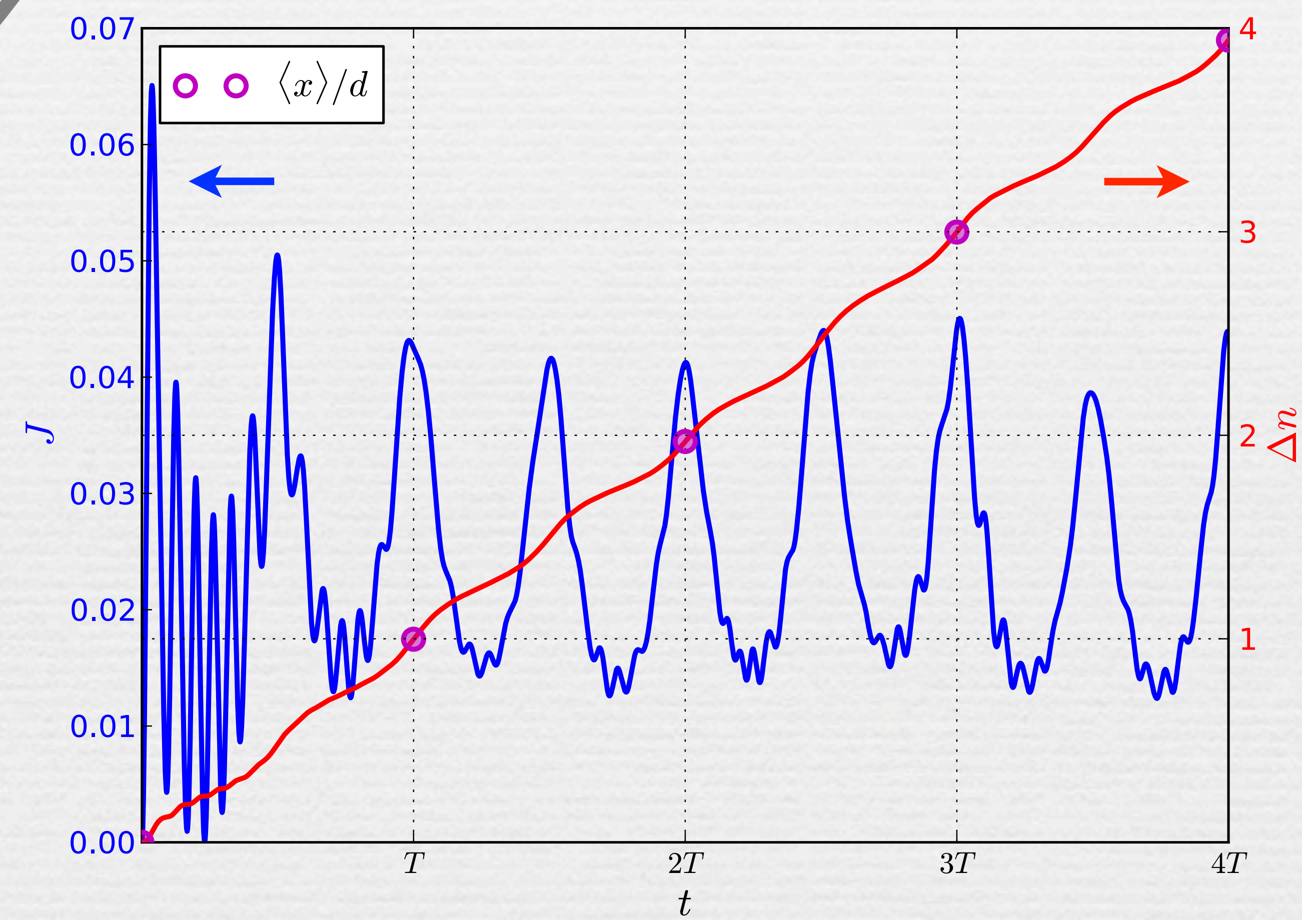

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Temperature & Non-adiabatic effect

$$\text{Temperature} \ll \frac{\Delta}{k_B} \qquad\qquad T \gg \frac{\hbar}{\Delta}$$

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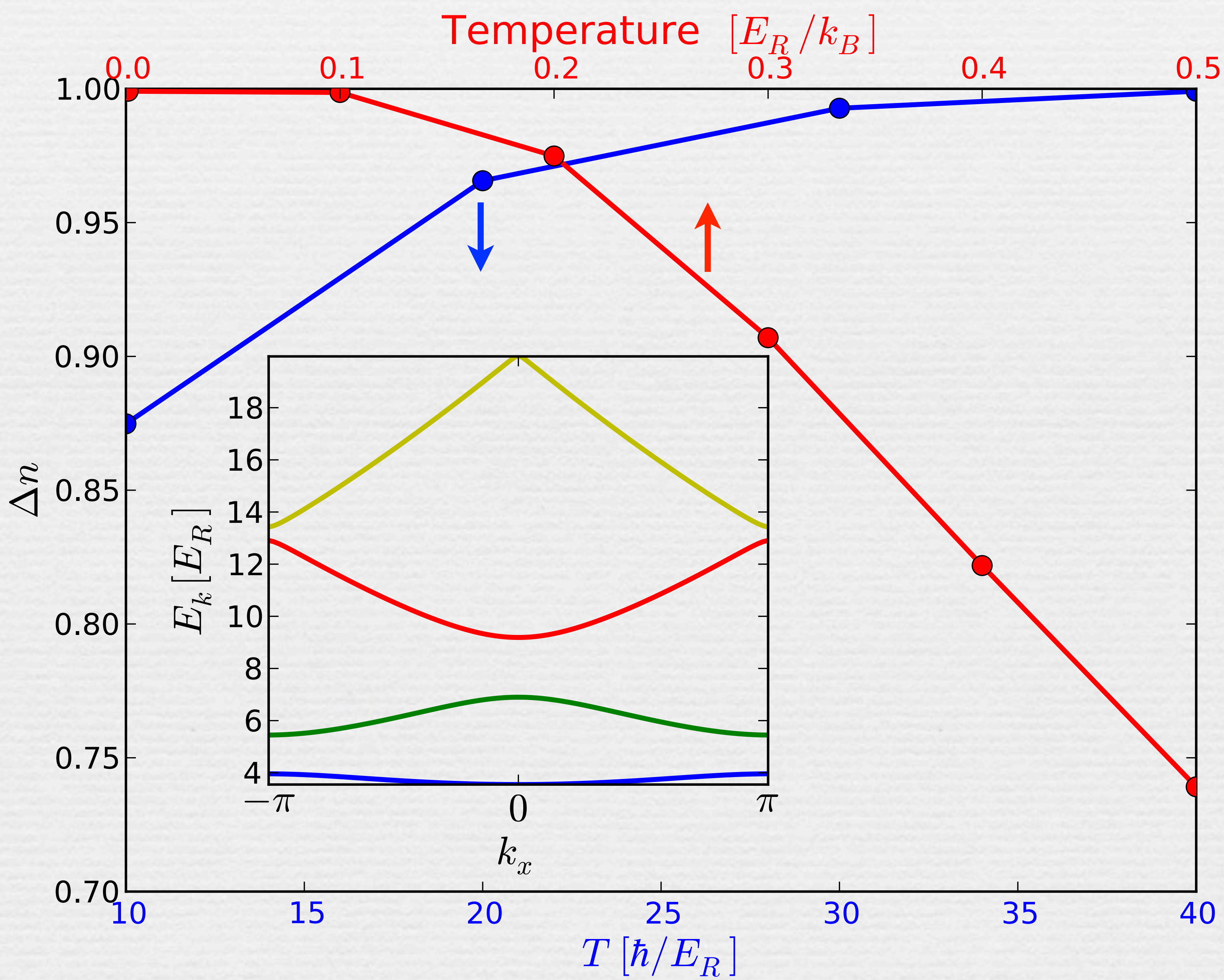


Measure Chern number of 2D optical lattice

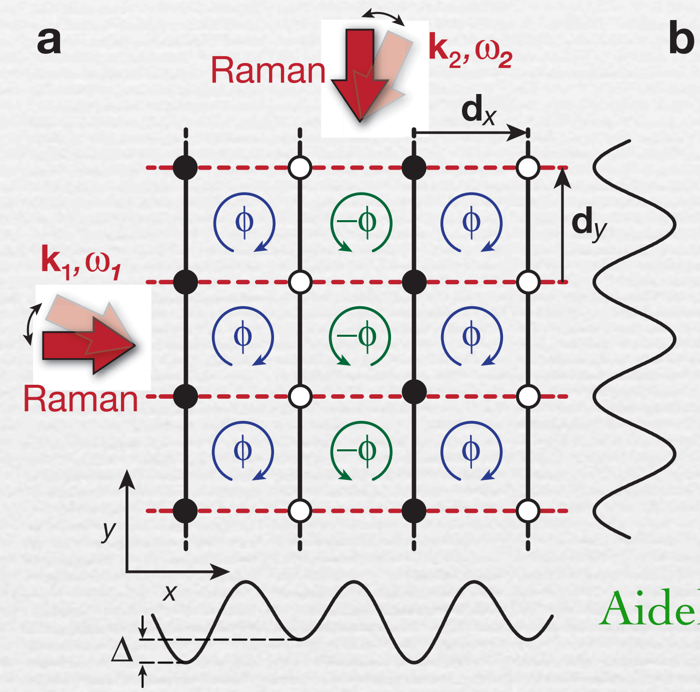
with

Topological charge pumping

Synthetic gauge-field in optical lattices

- Imprint complex phases to the hopping amplitude

- Staggered flux lattice Munich



Aidelsburger *et al*

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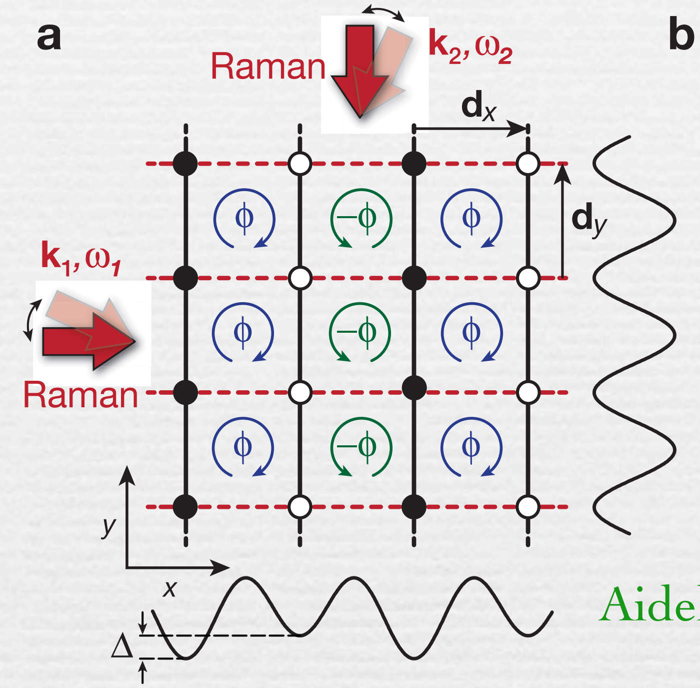
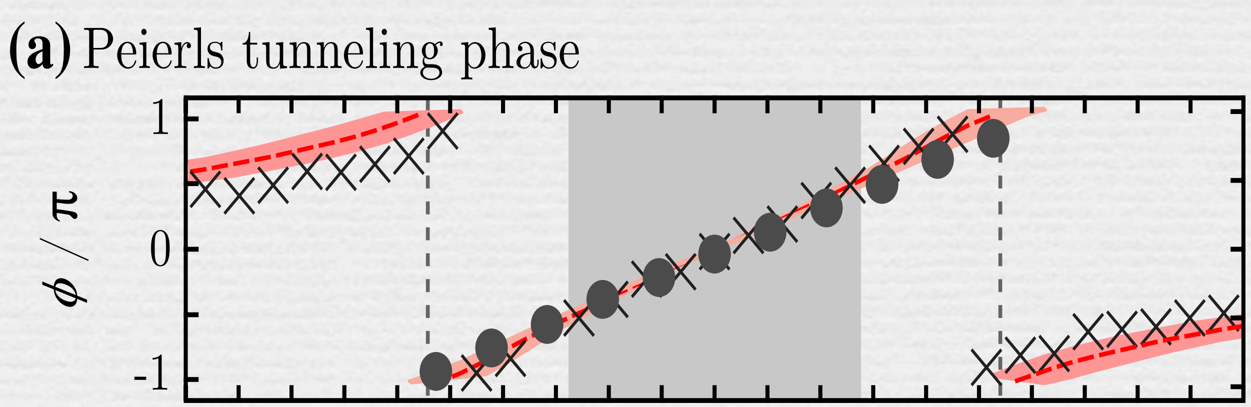


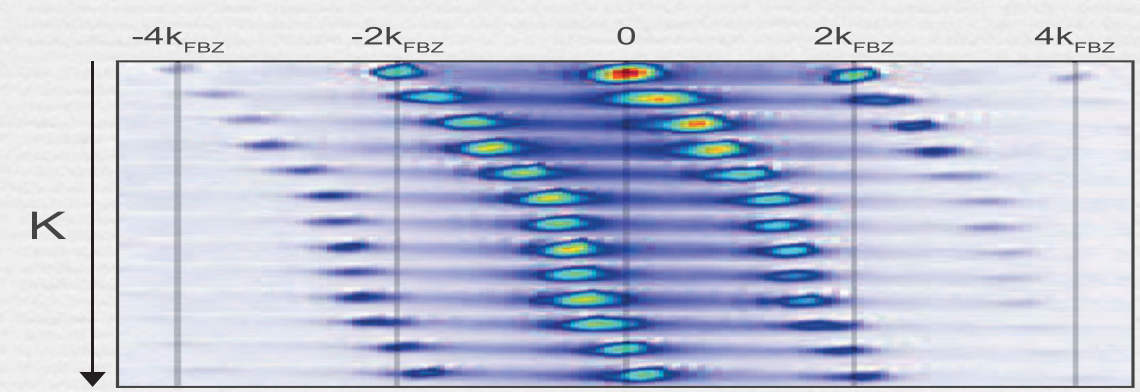
Aidelsburger *et al*

- 1D Peierls lattice NIST, Hamburg

$$H = -J \sum_m e^{i2\pi\Phi} c^\dagger_{m+1} c_m + H.c.$$



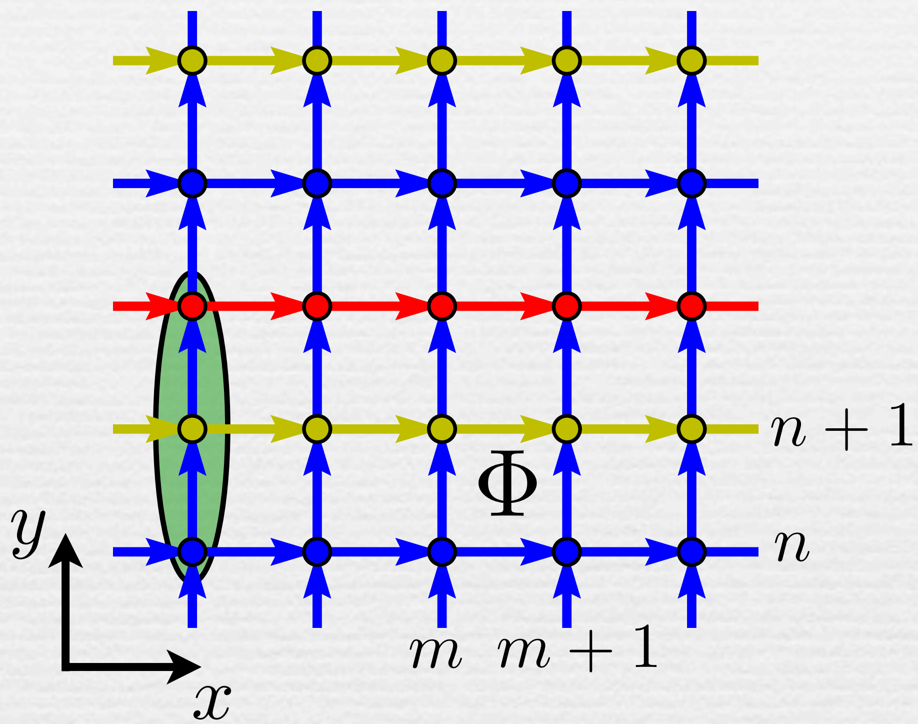
Jimenez-Garcıa *et al*



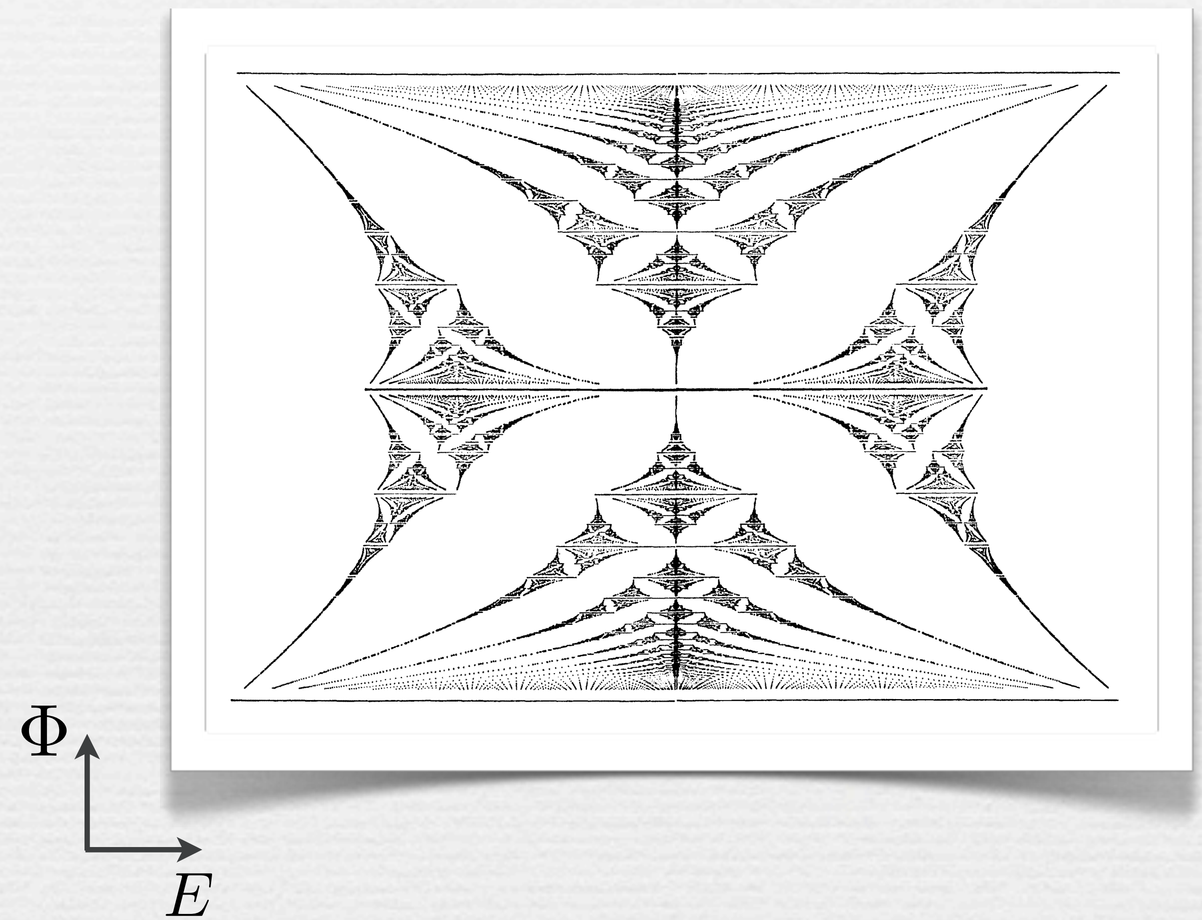
Struck *et al*

Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c^\dagger_{m+1,n} c_{m,n} + c^\dagger_{m,n+1} c_{m,n} + H.c. \quad \Phi = p/q$$



Hofstadter, 1976



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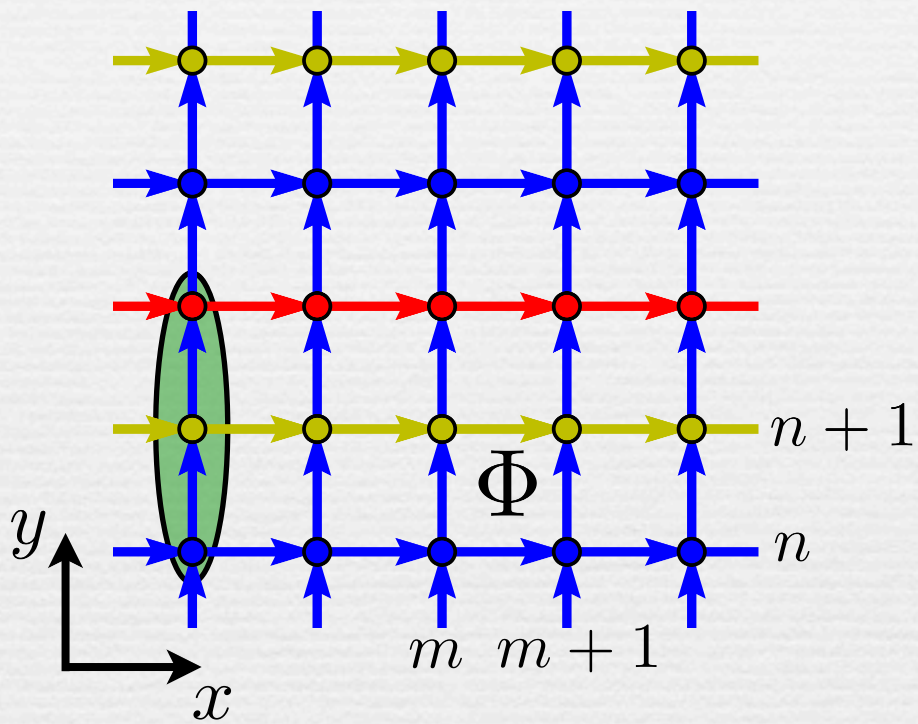

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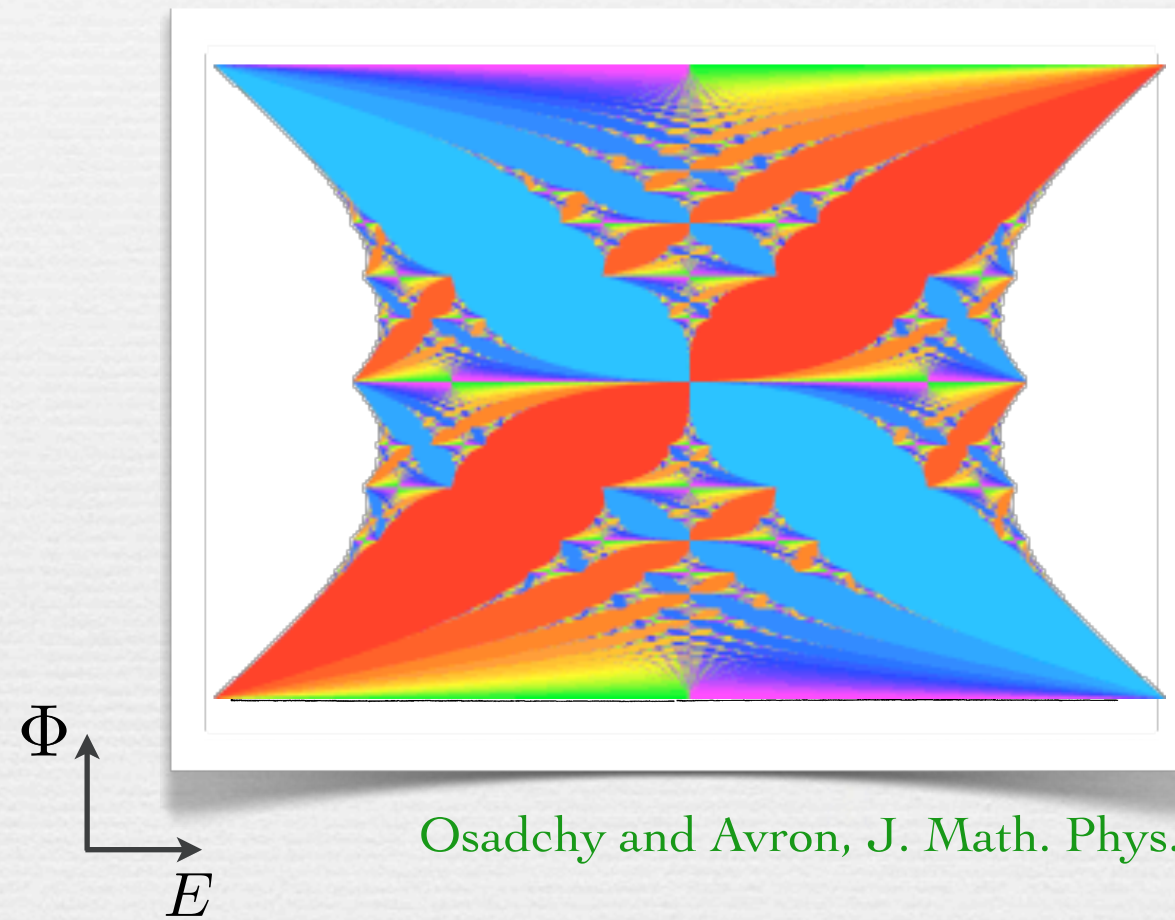

Osadchy and Avron, J. Math. Phys. 2001

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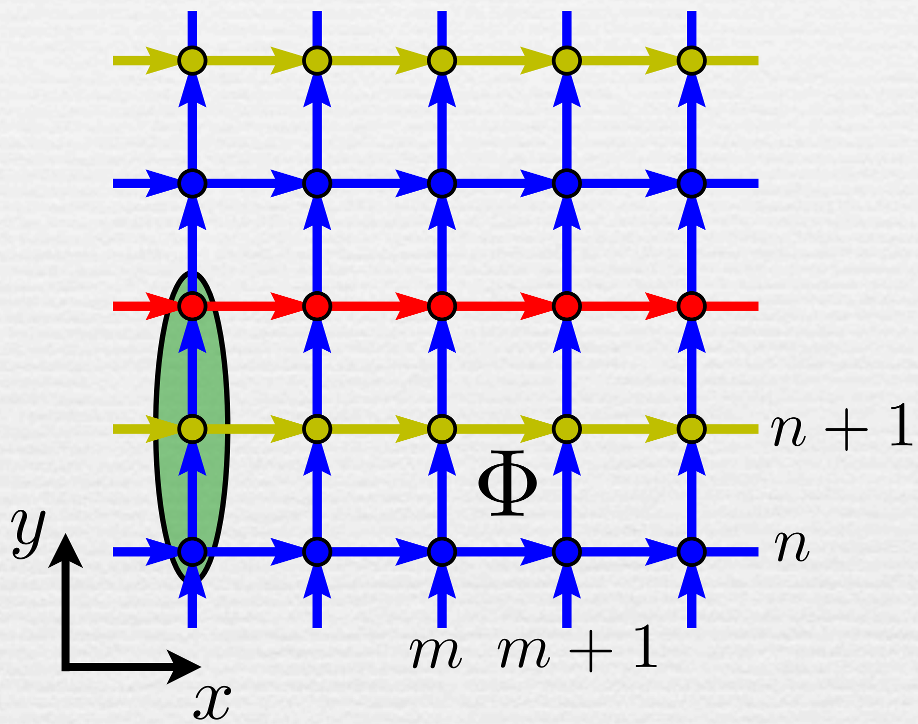

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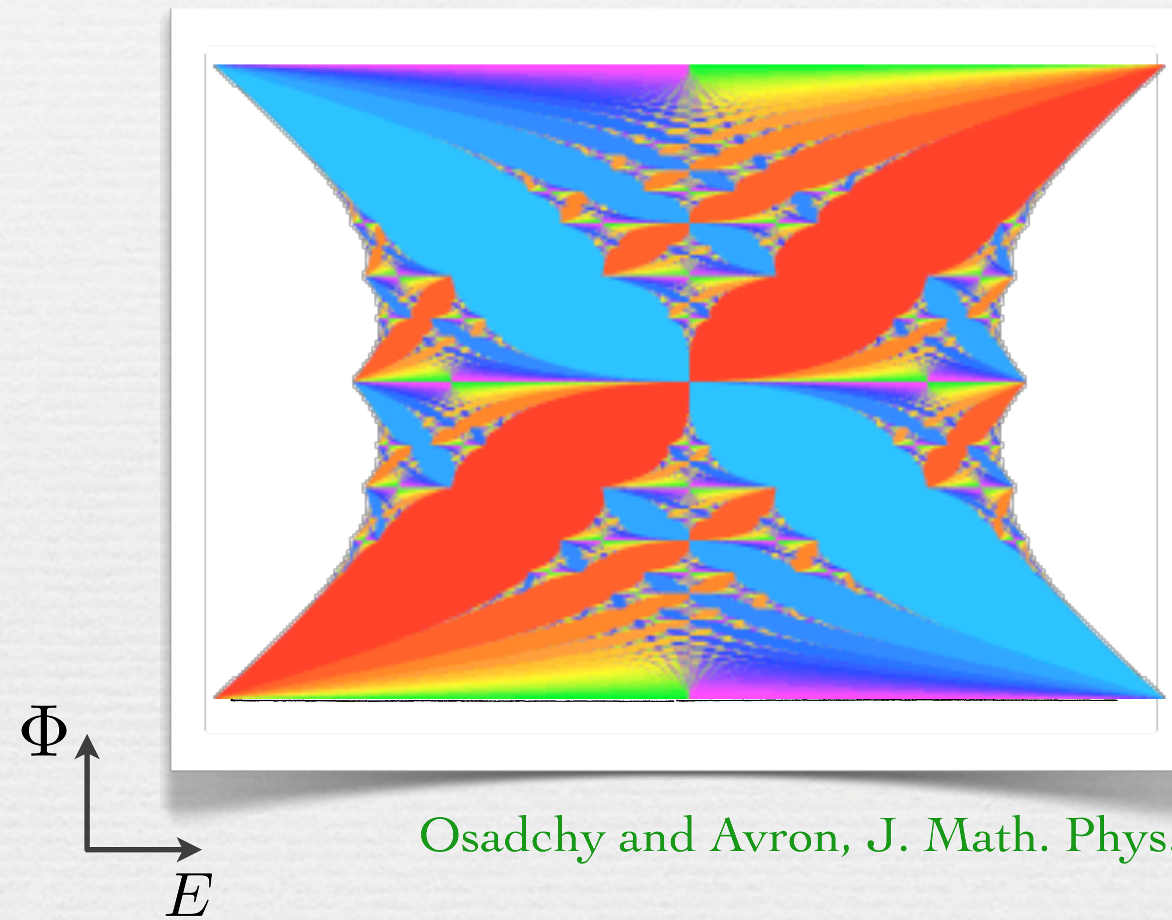
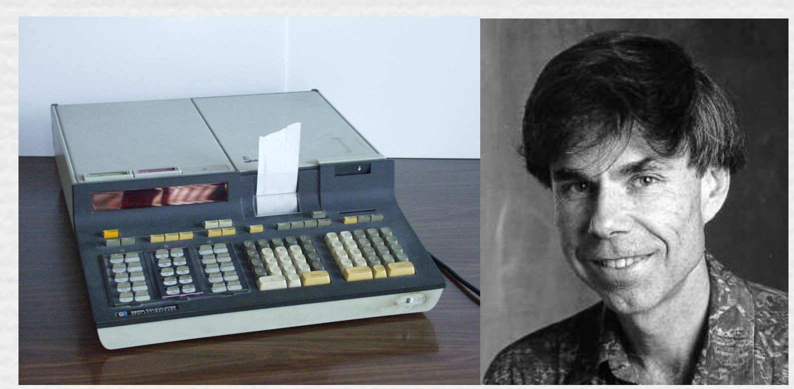


Osadchy and Avron, J. Math. Phys. 2001

NO sharp edge states in harmonic trapping potential

Buchhold *et al*



How to measure Chern # ?

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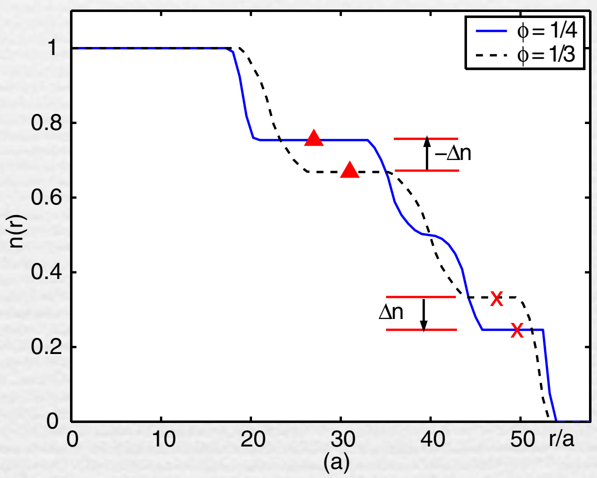
Density profile

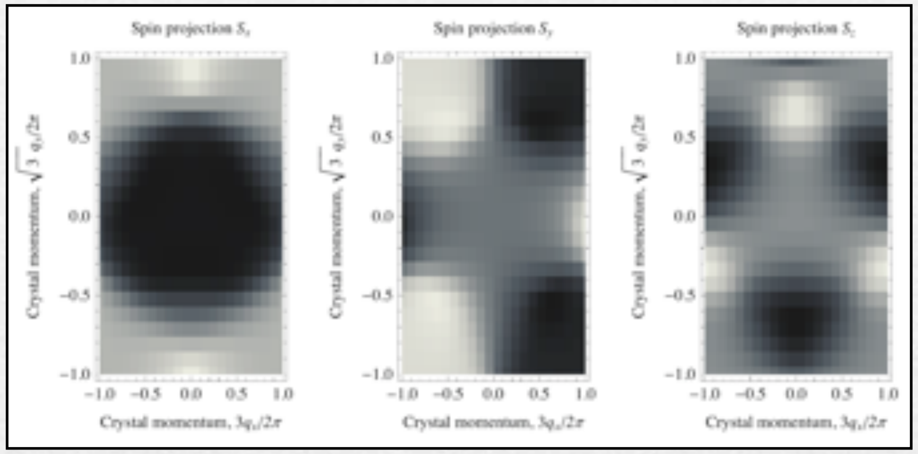
Umucalilar *et al*



How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



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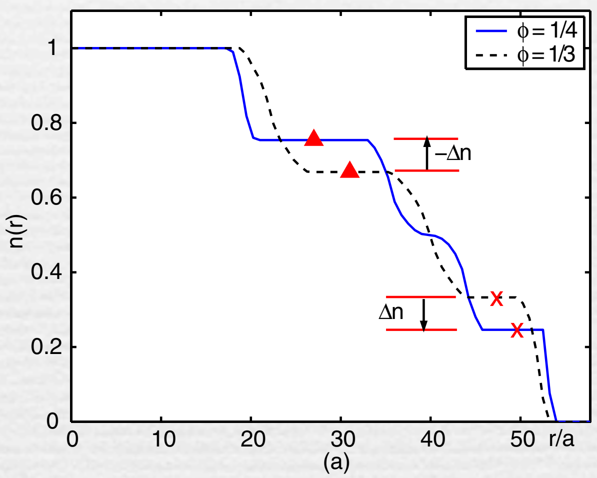

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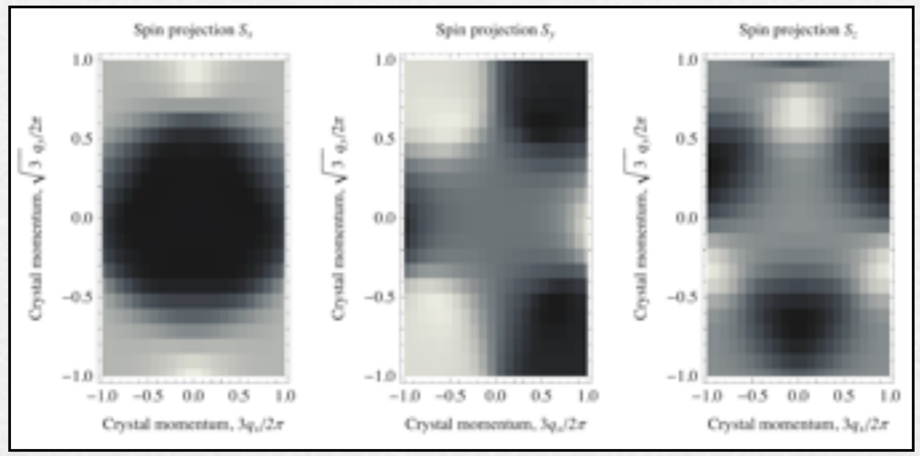

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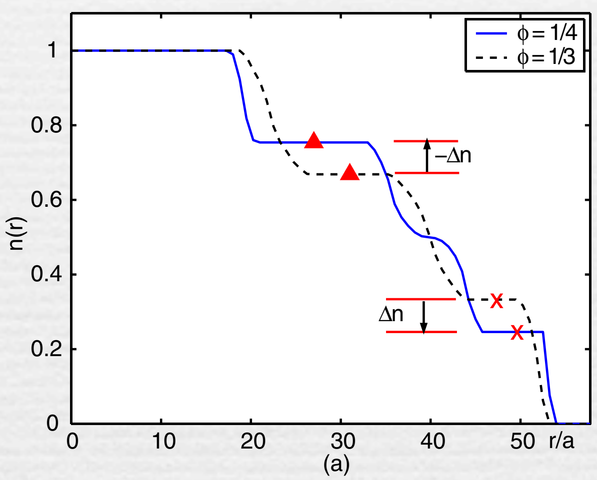
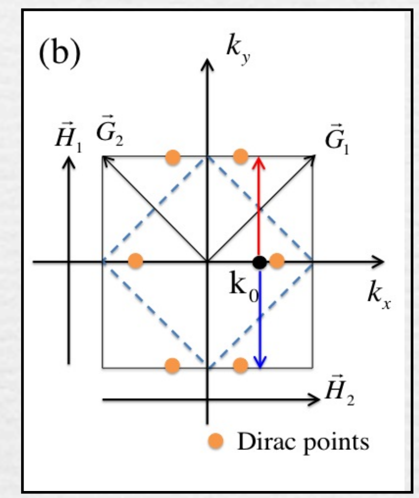


Zak phases

Abanin *et al*



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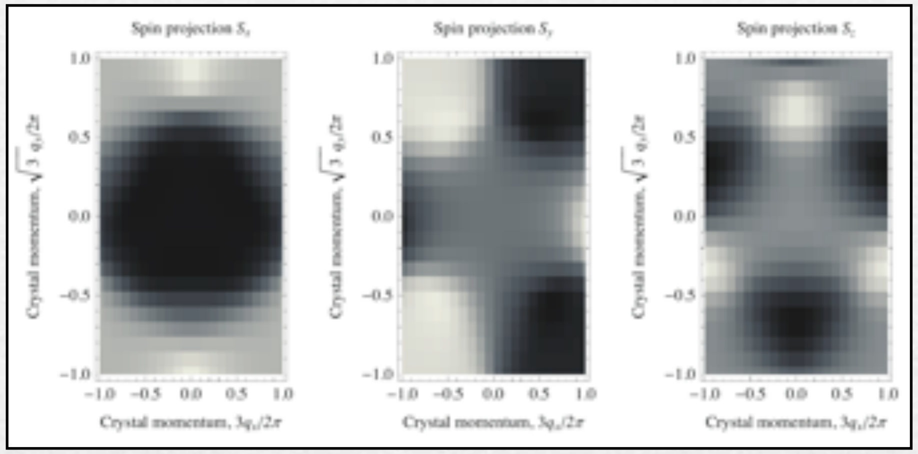

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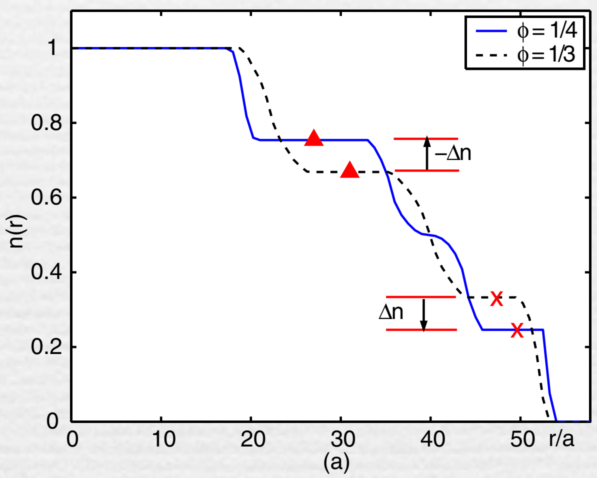

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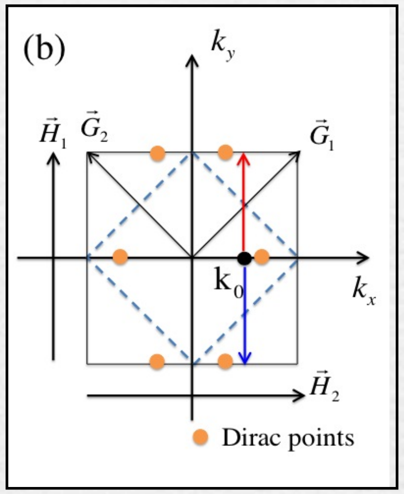

Semi-classical dynamics

Price *et al*

$$\dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} - \frac{\mathbf{F}}{\hbar} \times \hat{\mathbf{z}}\Omega(\mathbf{k})$$

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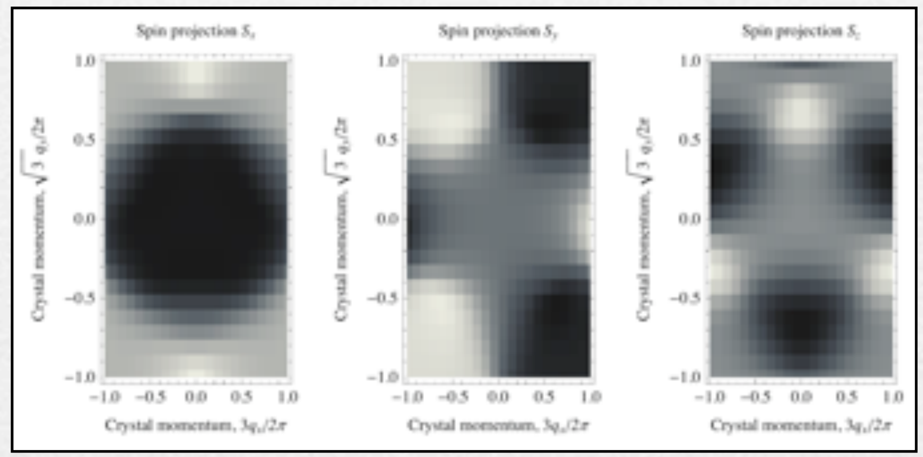

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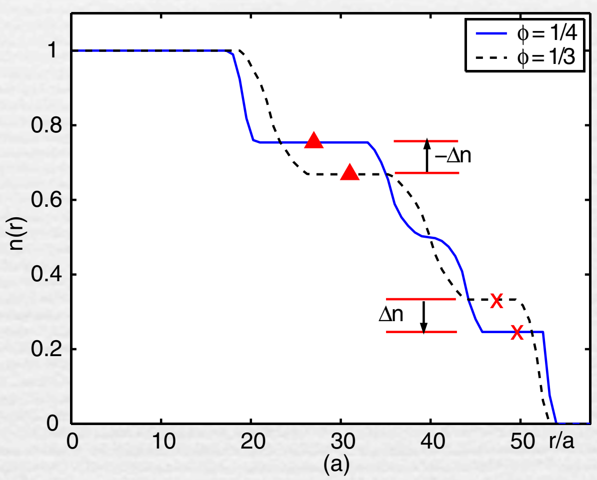

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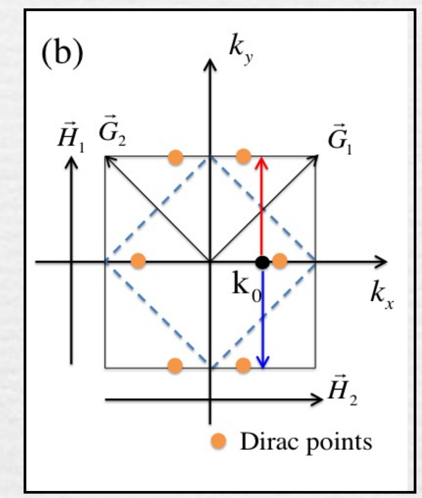

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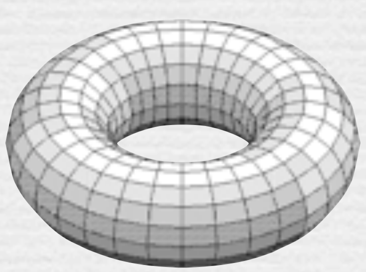


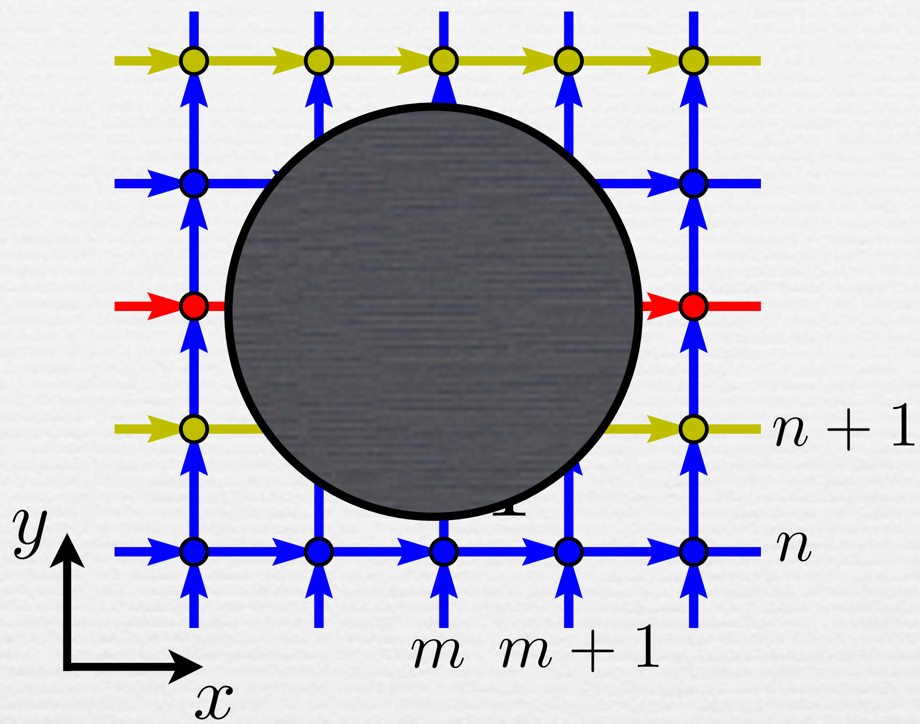
We propose a new probe based on

Topological Pumping Effect

$$\rho(k_x, y)$$

Hybrid time-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802



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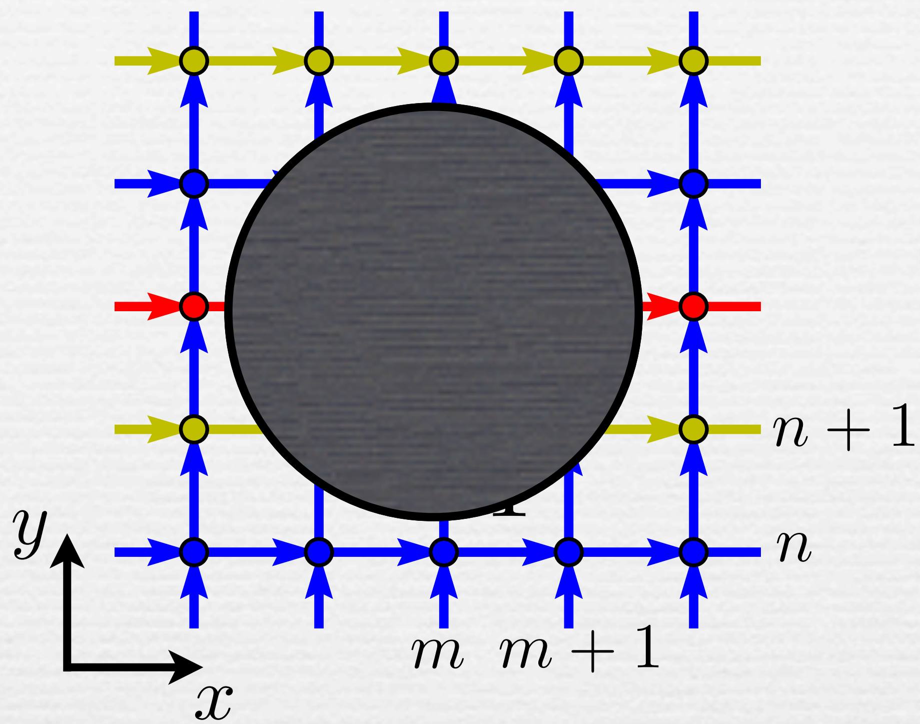

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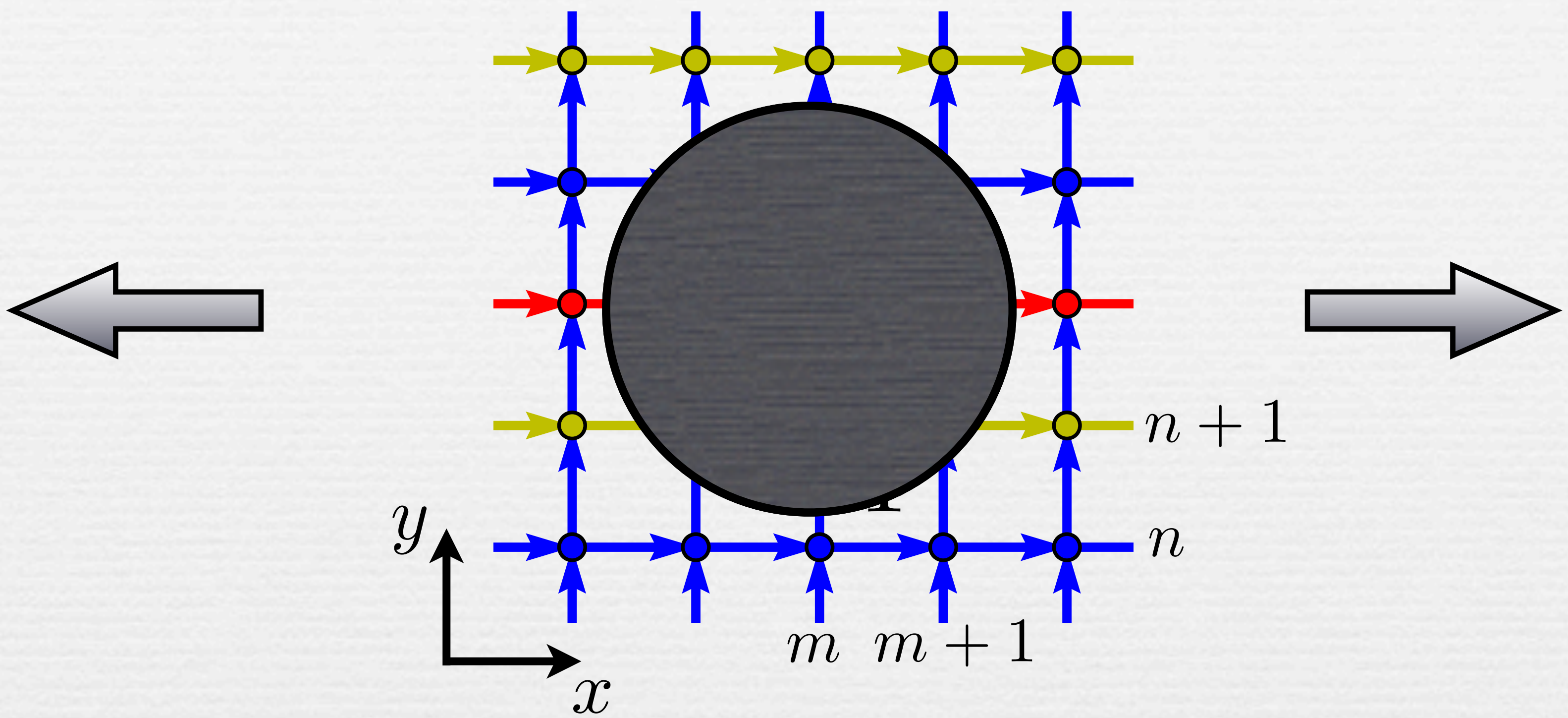

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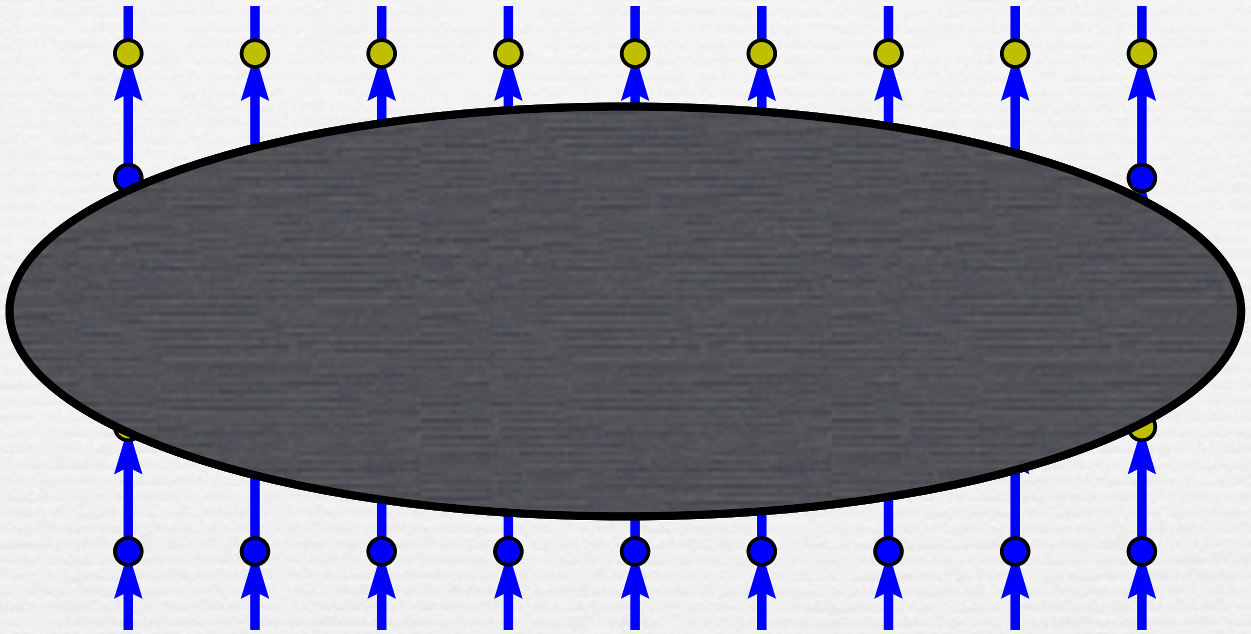

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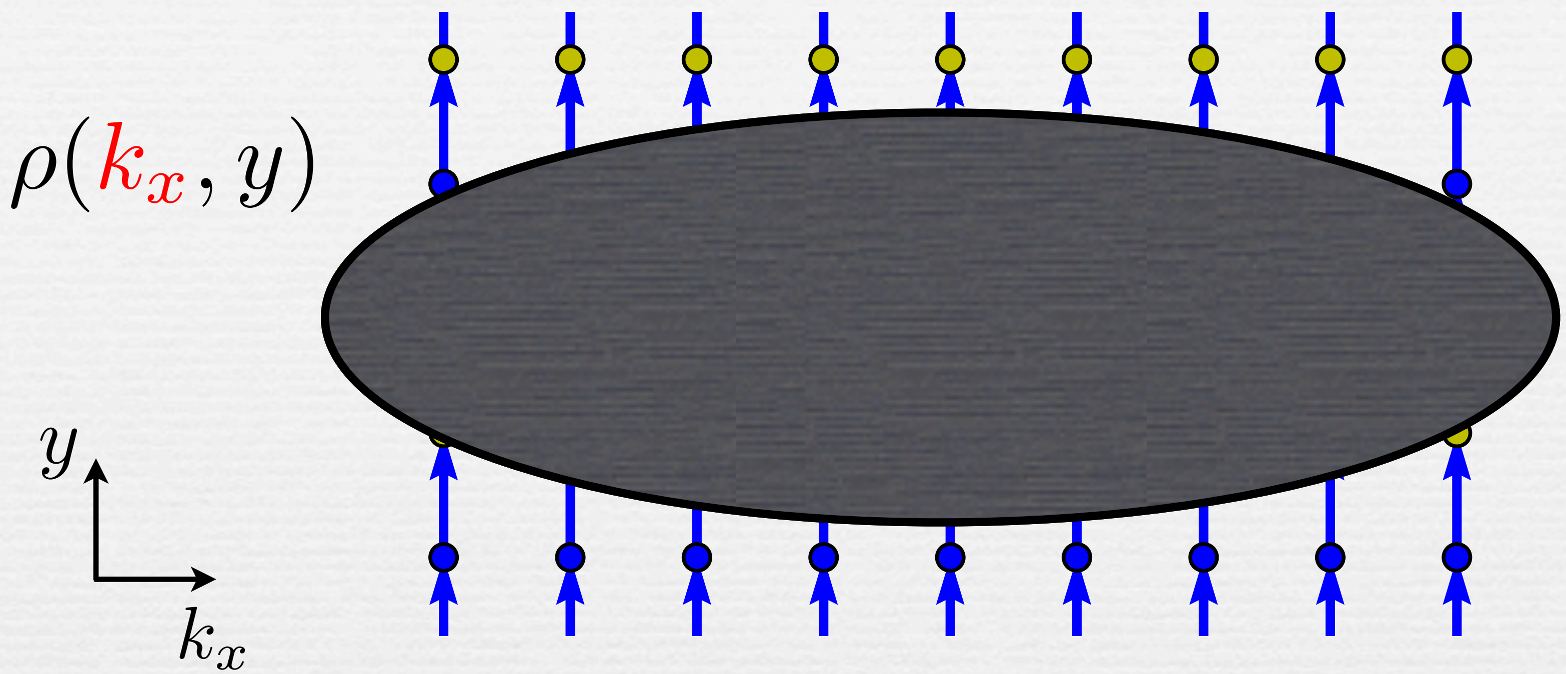

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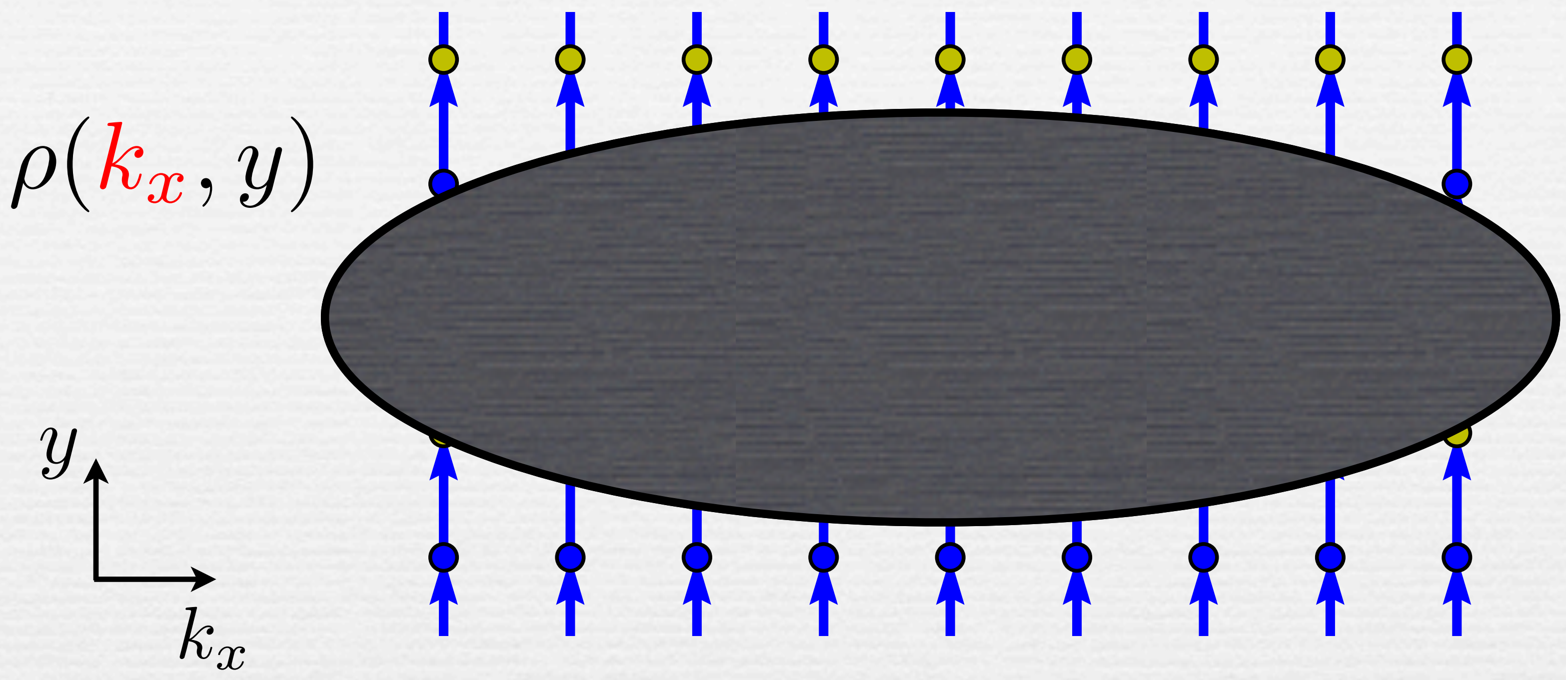
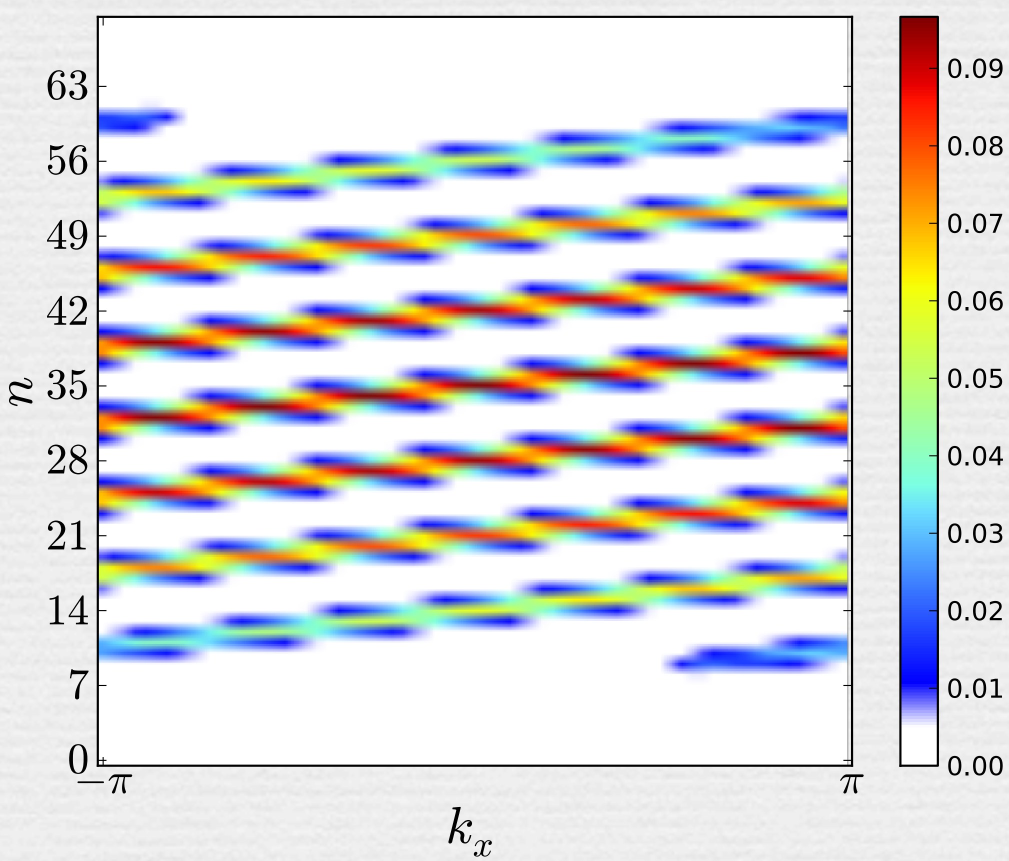


$\Phi = 1/7 \quad C = 1$



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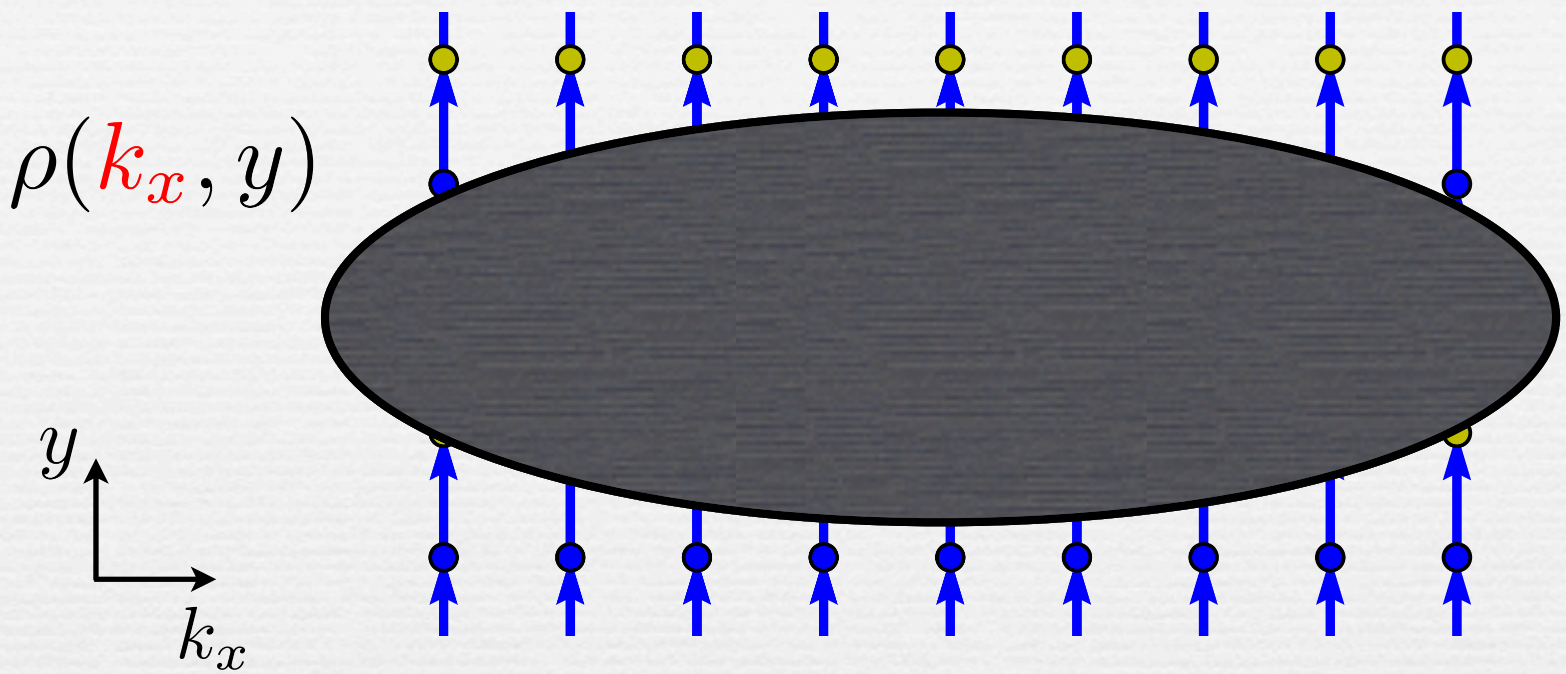

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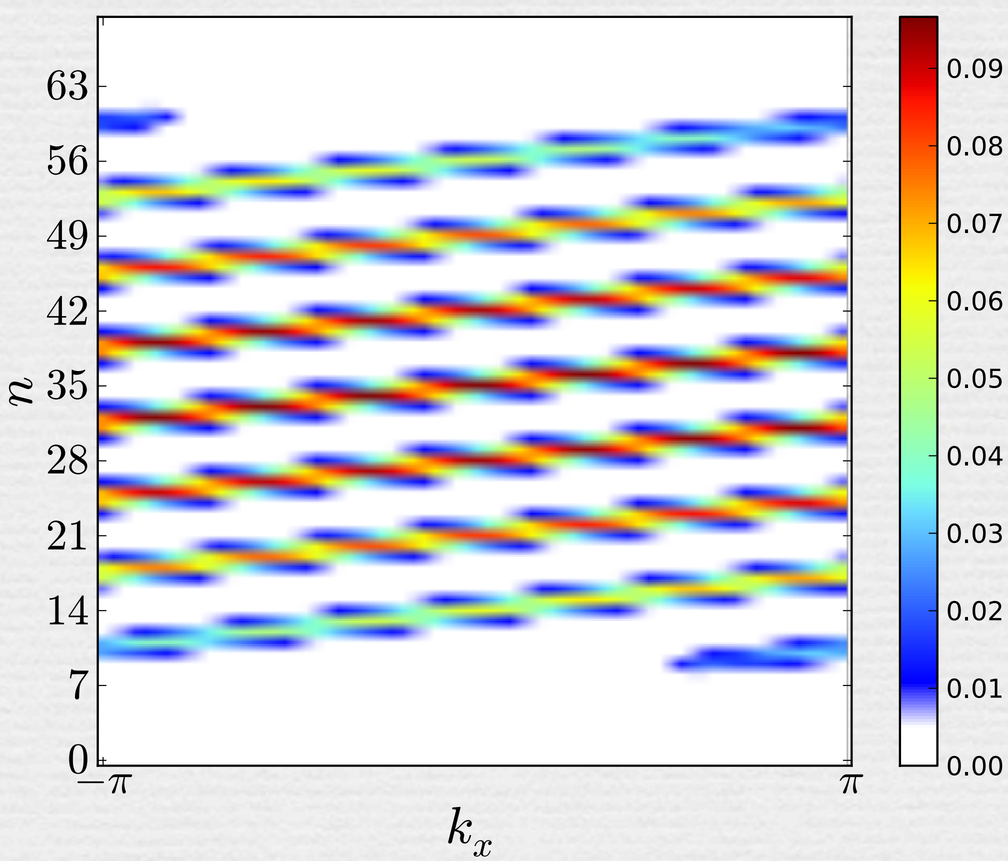
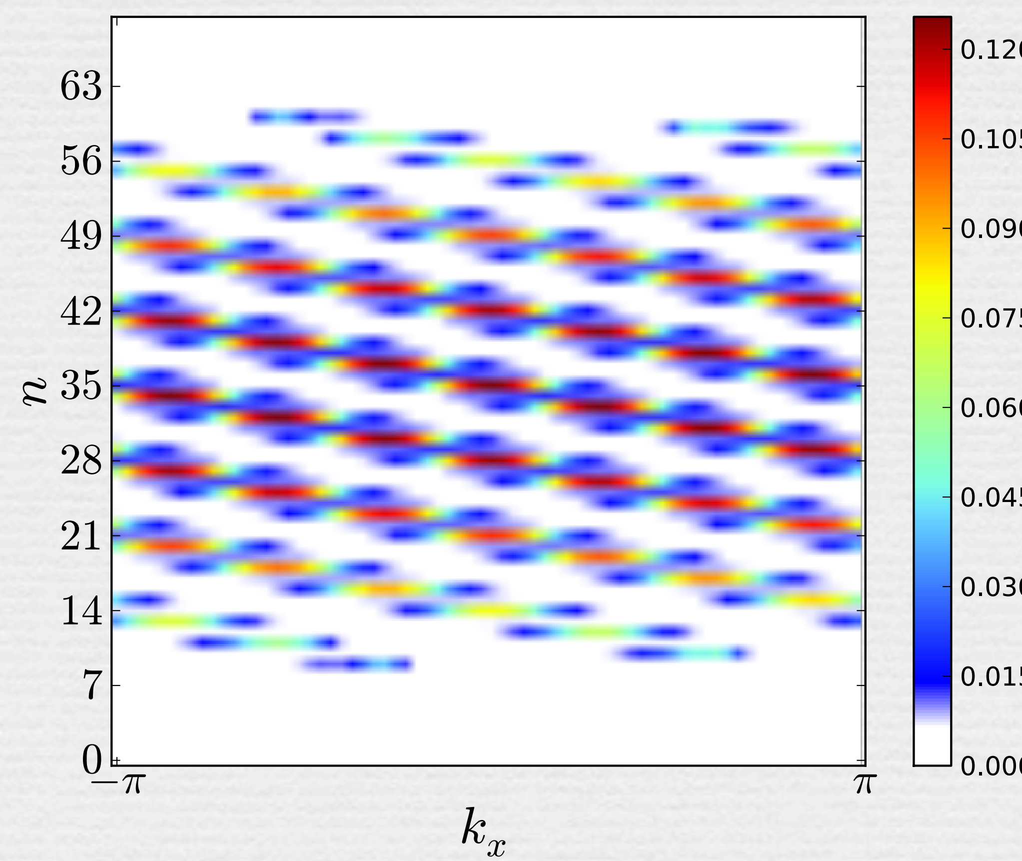
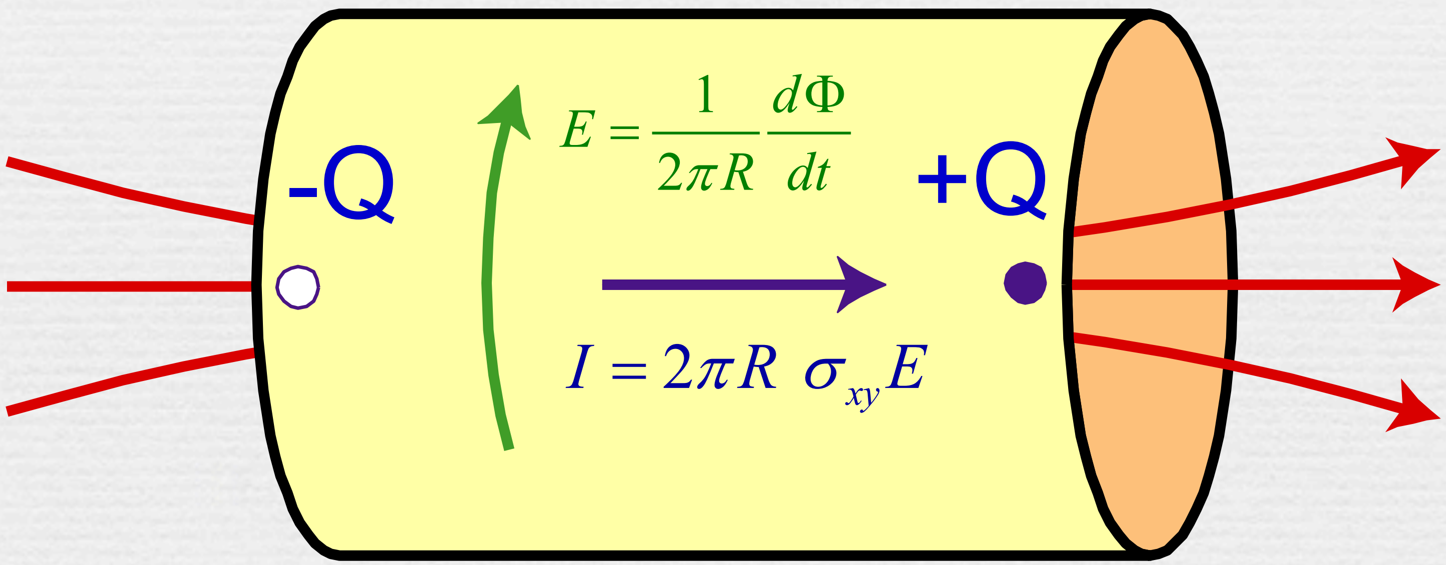


$\Phi = 3/7 \quad C = -2$



Why it works?

Topological charge pumping



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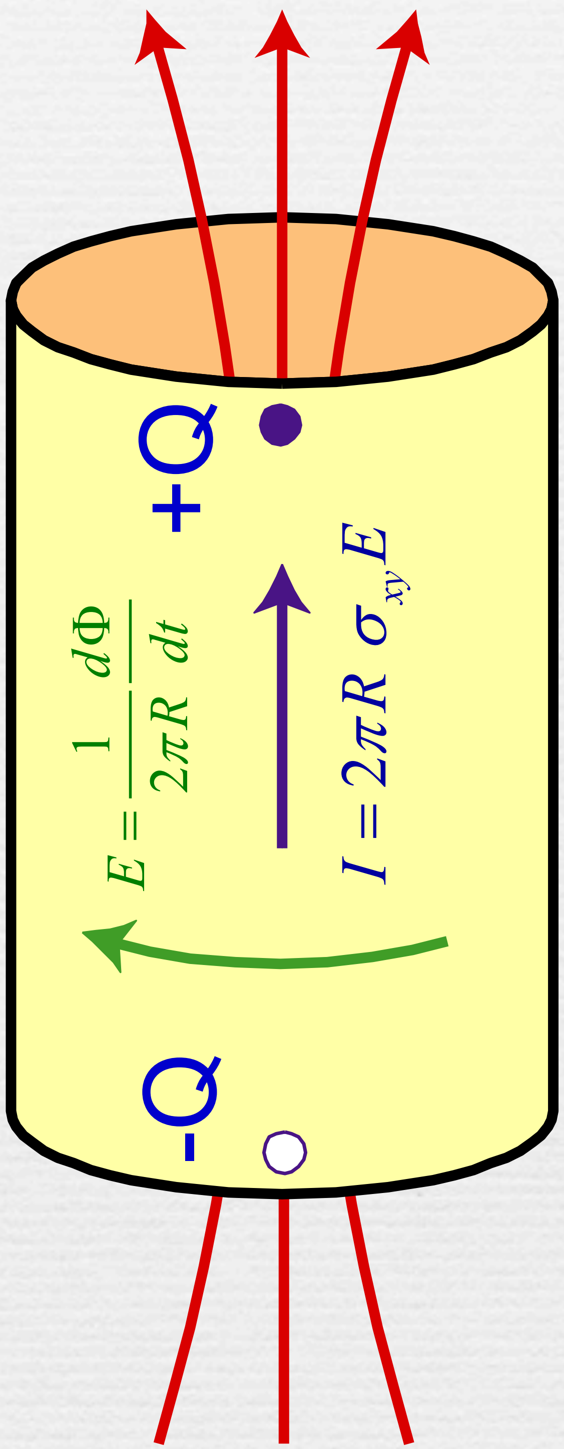

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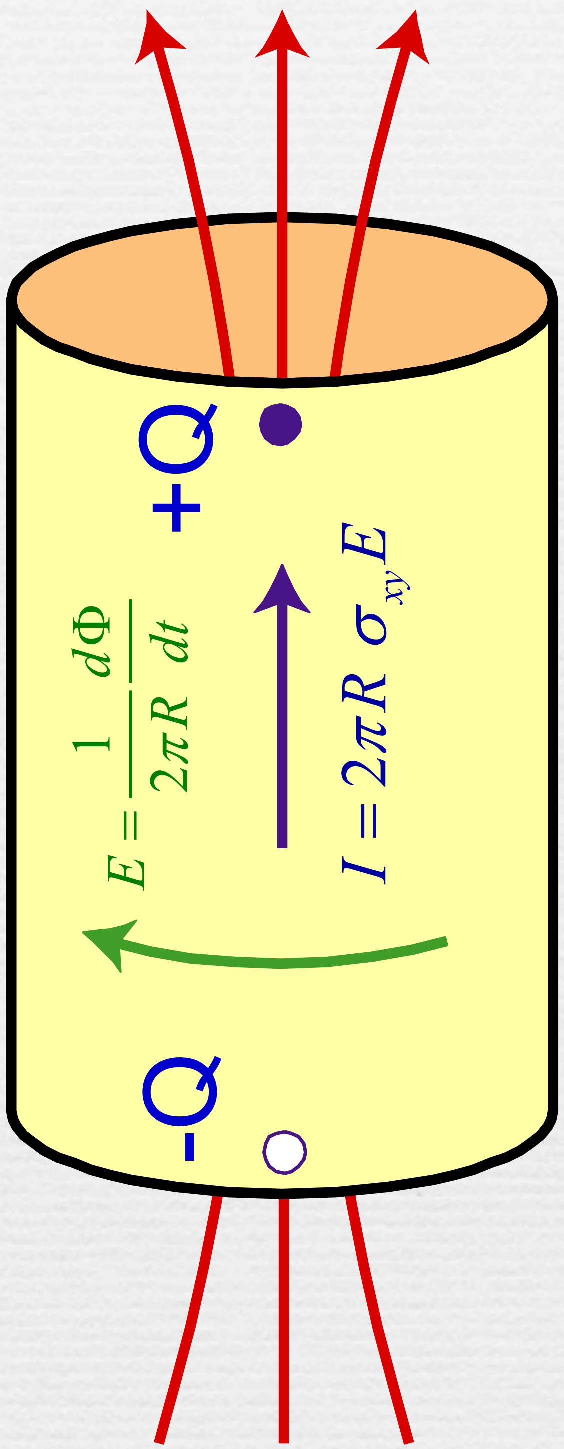

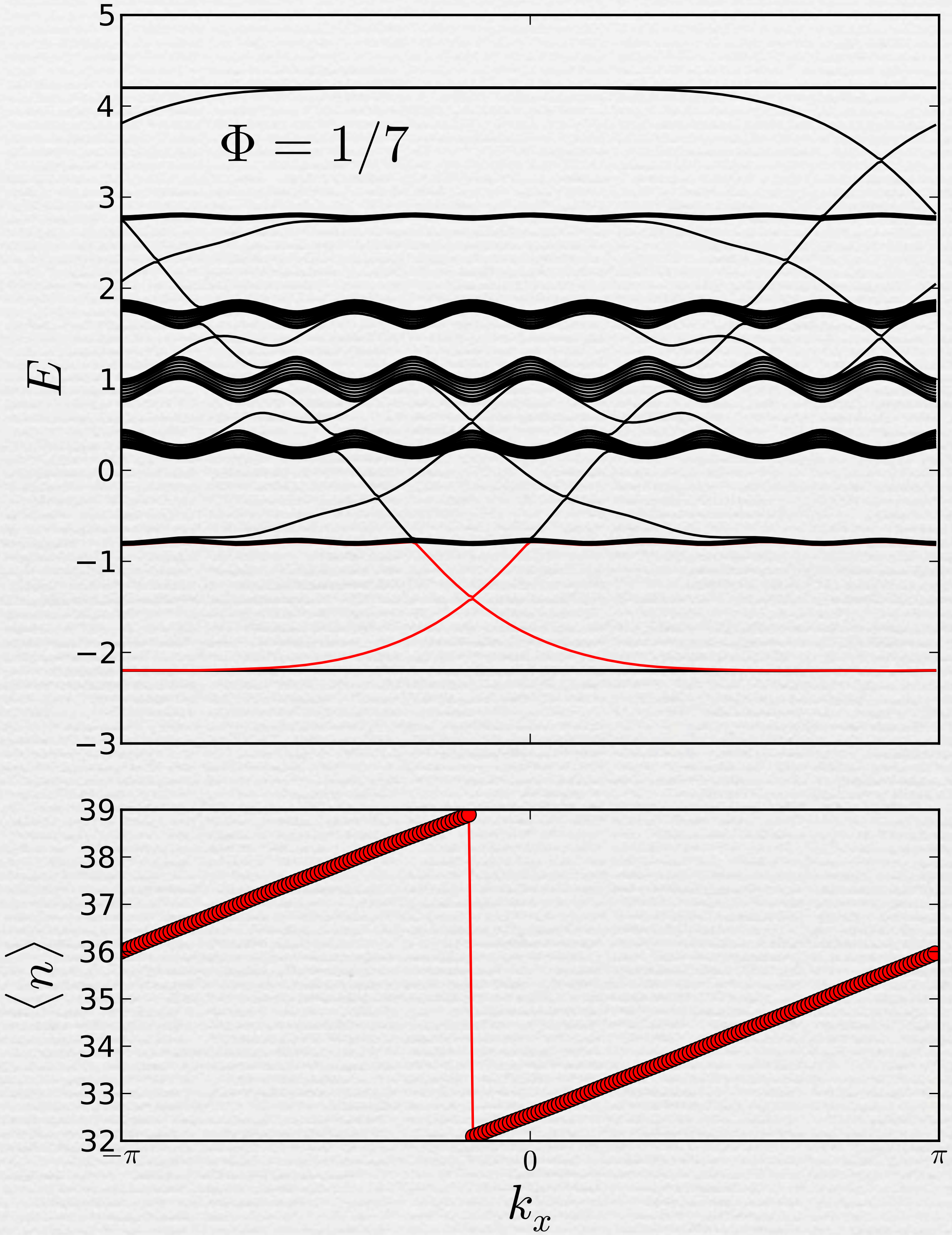

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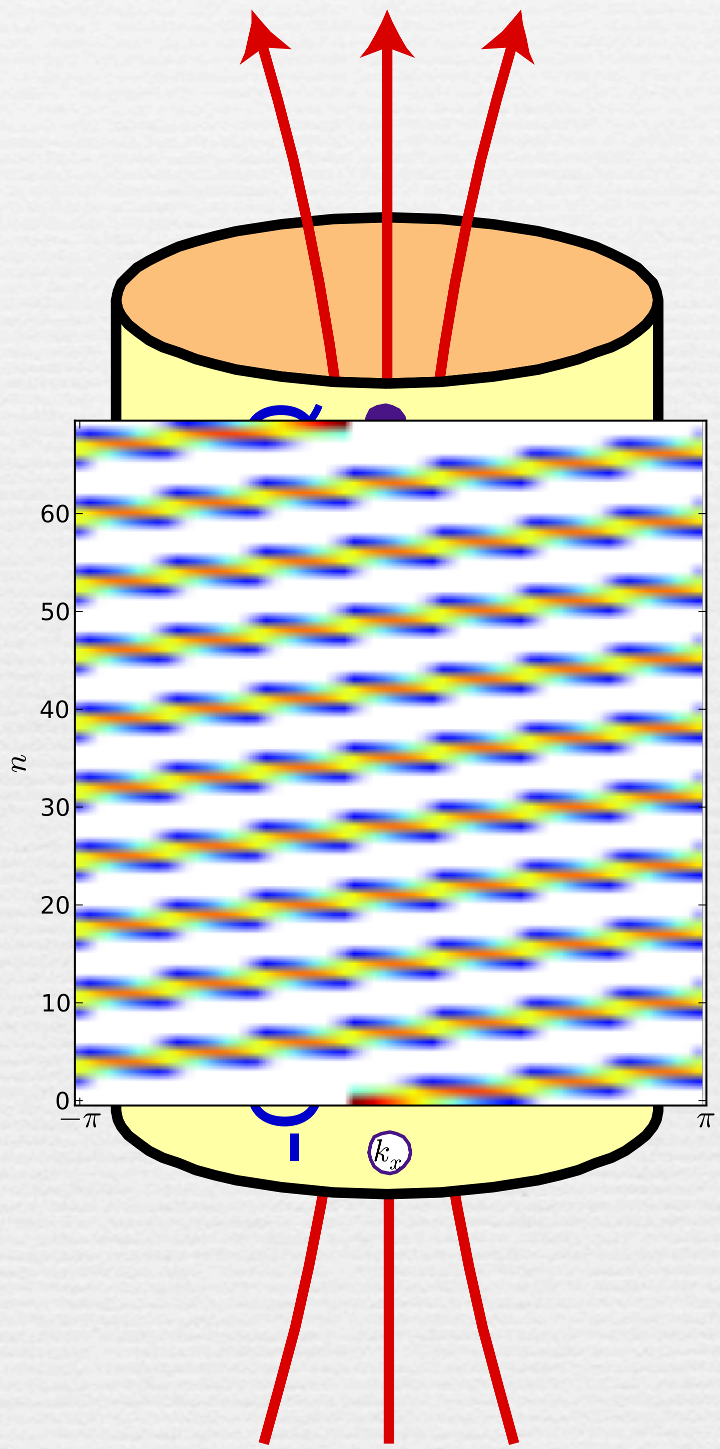
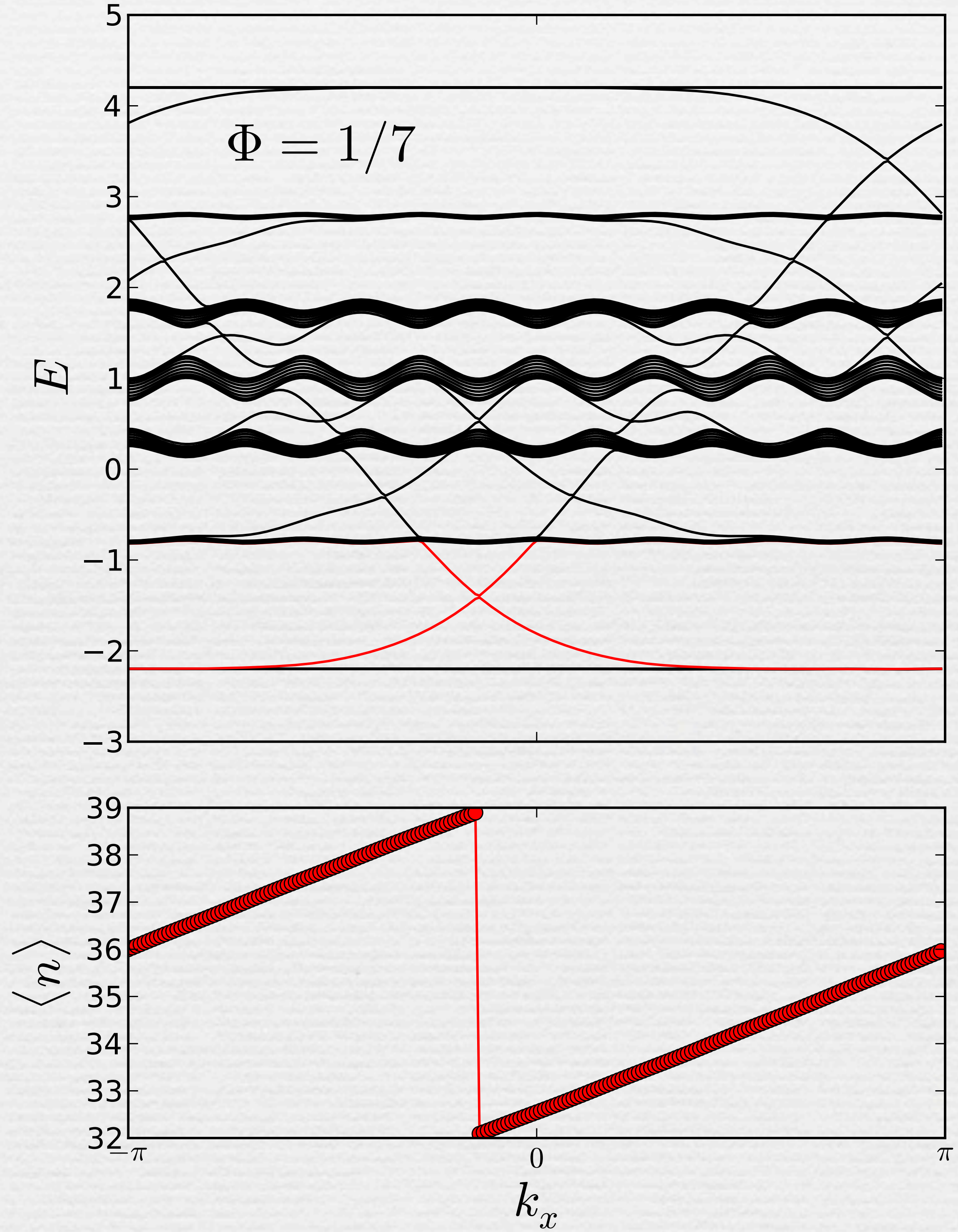
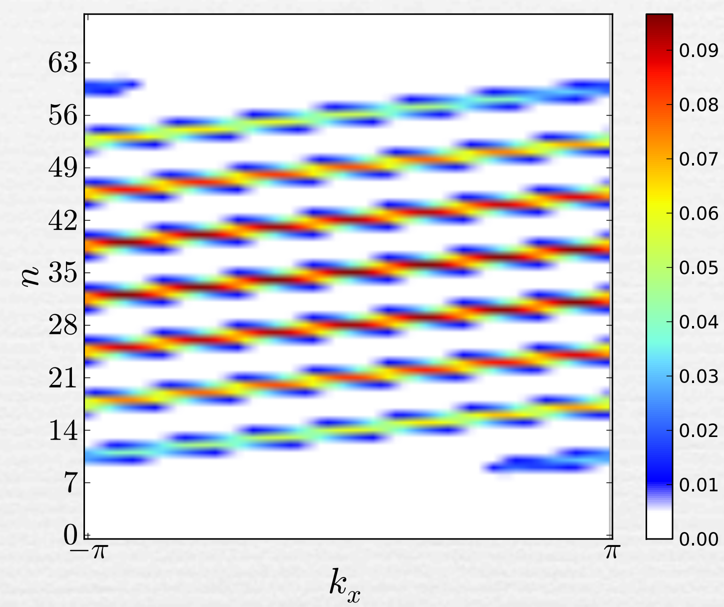




Quantitative Characterizations

- Slope
- # of cuts (edge modes)
- COM along y-direction
- Bi-partition number of particle (trace index)

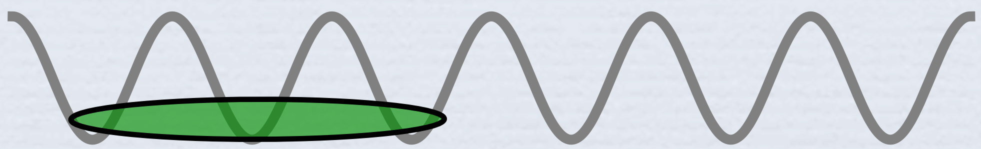


Salient features

- Bulk detection, does not require edge states
- $\rho(k_x, y)$ is nearly impossible to measure in solids, but accessible to cold atom toolbox
- Can be extended to interacting case

Fractional charge pumping

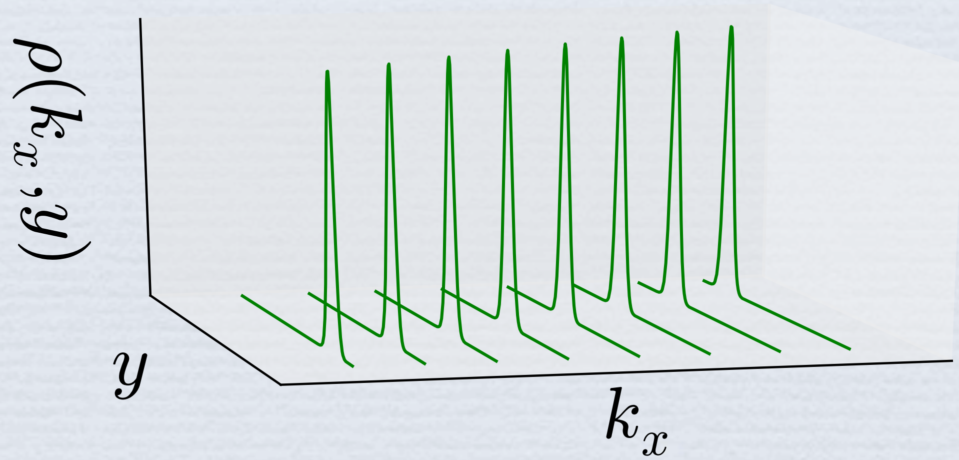
1D lattice



Interaction is crucial
for opening an energy gap

2D Laughlin state

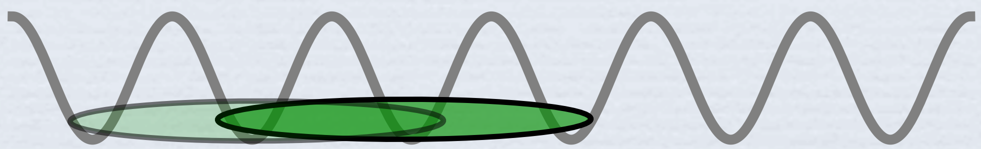
$$\rho(k_x, y) = \frac{\nu}{\sqrt{\pi}} e^{-(y-k_x)^2}$$



Can be used to detect FQHE and fractional Chern insulators realized in optical lattice Cooper *et al*, Yao *et al*, Nielsen *et al*

Fractional charge pumping

1D lattice



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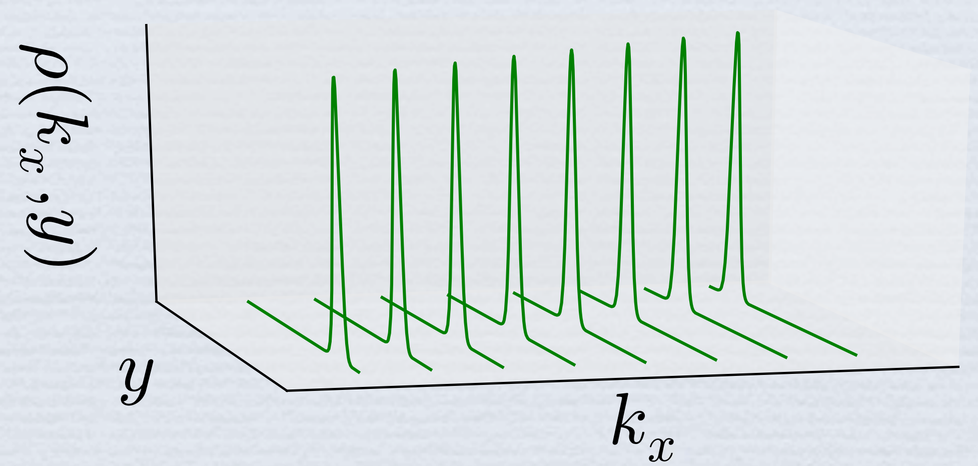

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Laughlin states on lattice

$$\Phi_{\text{Laughlin}} = \Phi_{\text{Hofstadter}}^{1/\nu}$$

Laughlin states on lattice

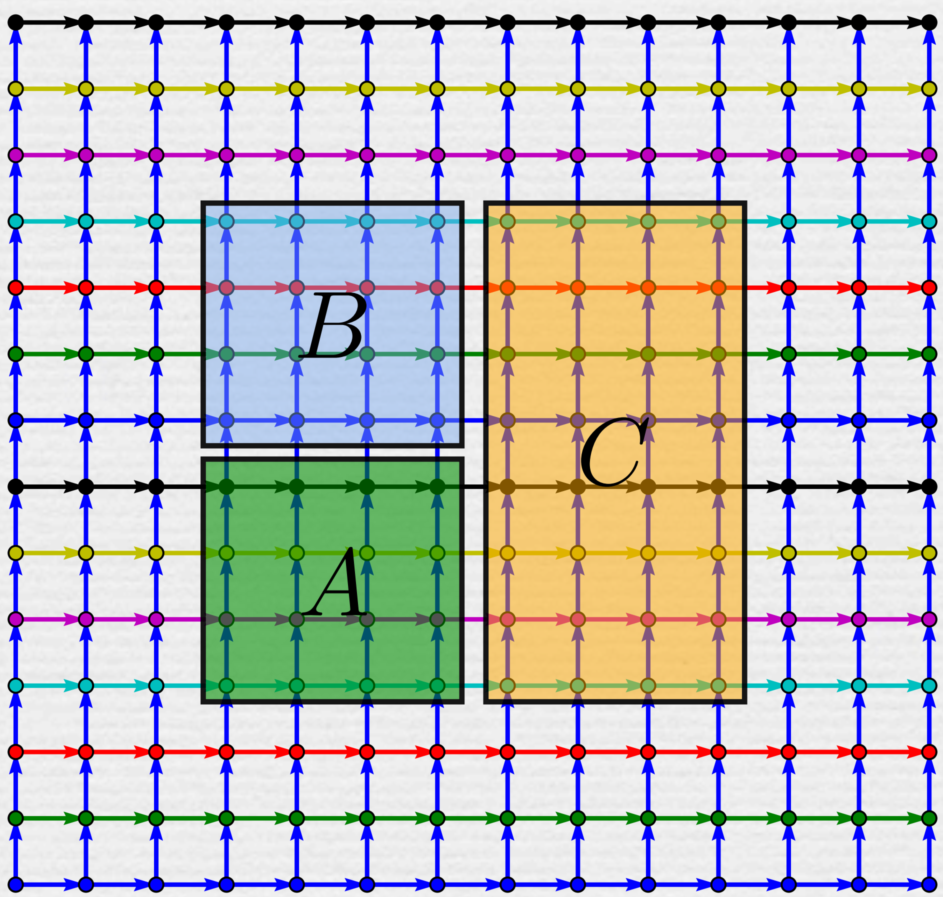
$$\boxed{\Phi_{\text{Laughlin}} = \Phi_{\text{Hofstadter}}^{1/\nu}}$$

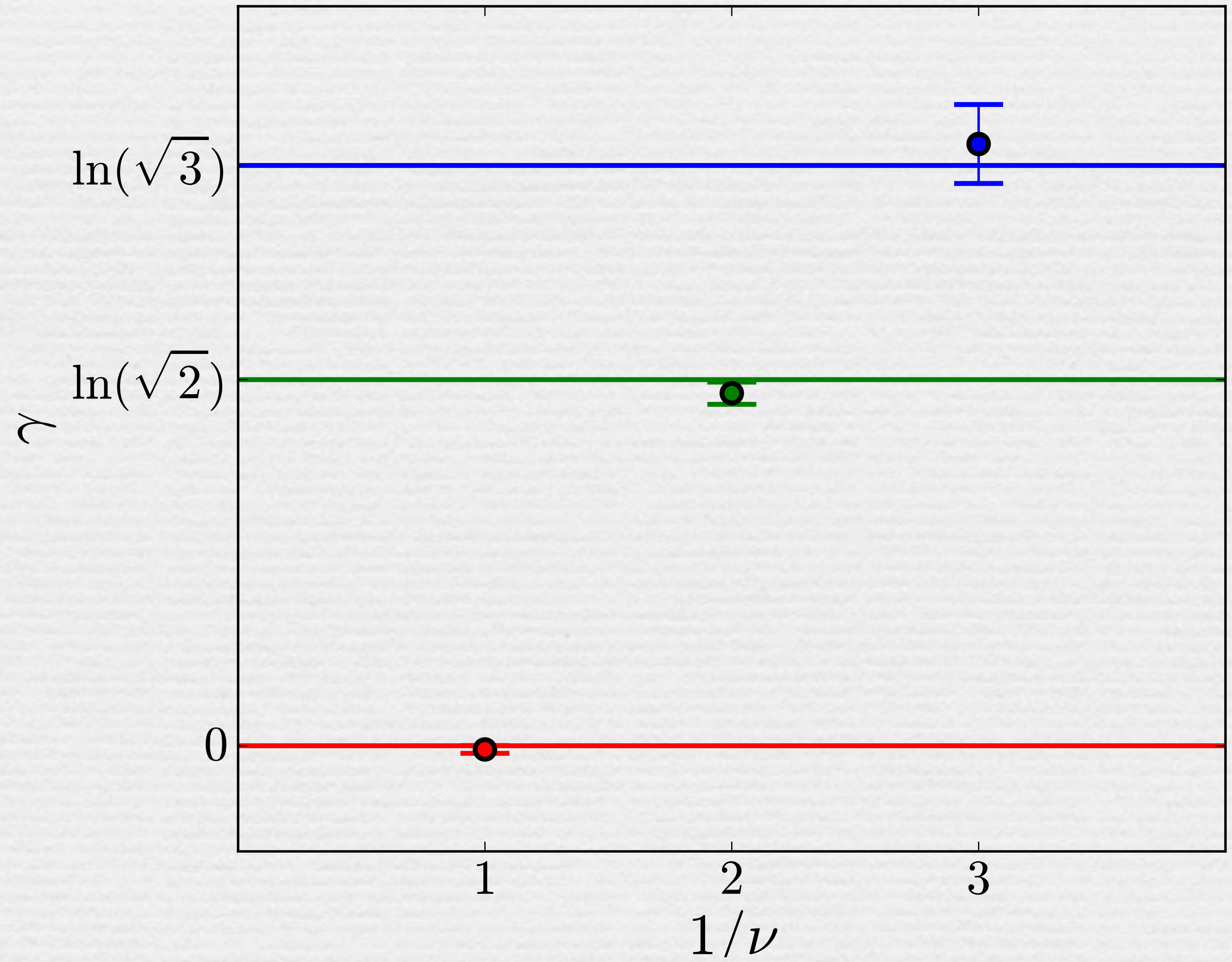
Topological entanglement entropy

Preskill, Kitaev, Levin, Wen

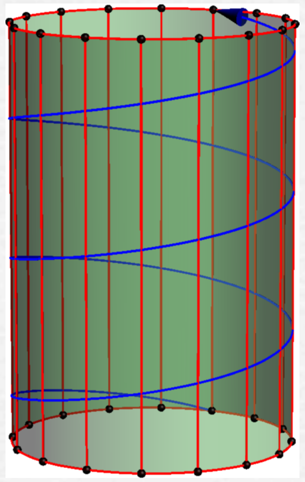
Zhang *et al*

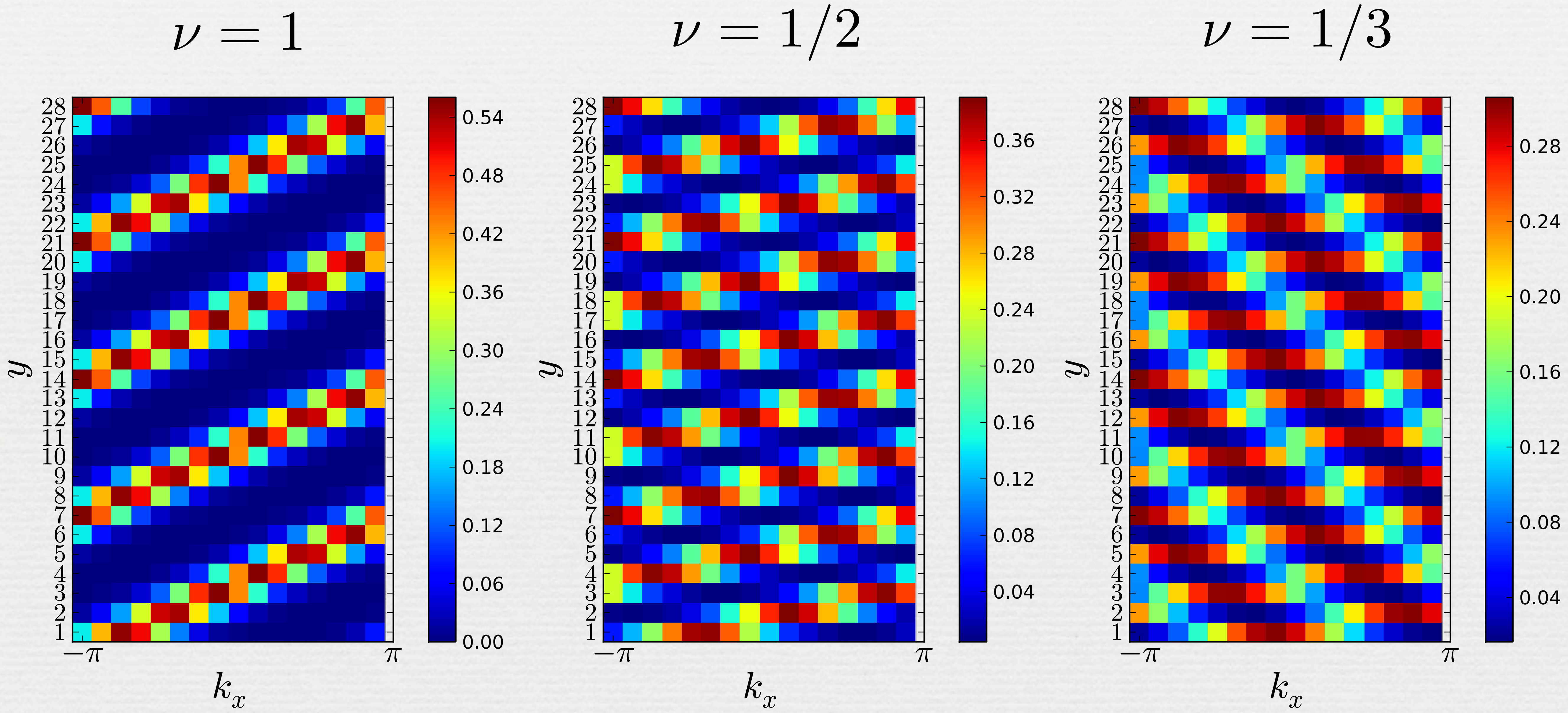
$$-\gamma = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$





Hybrid densities

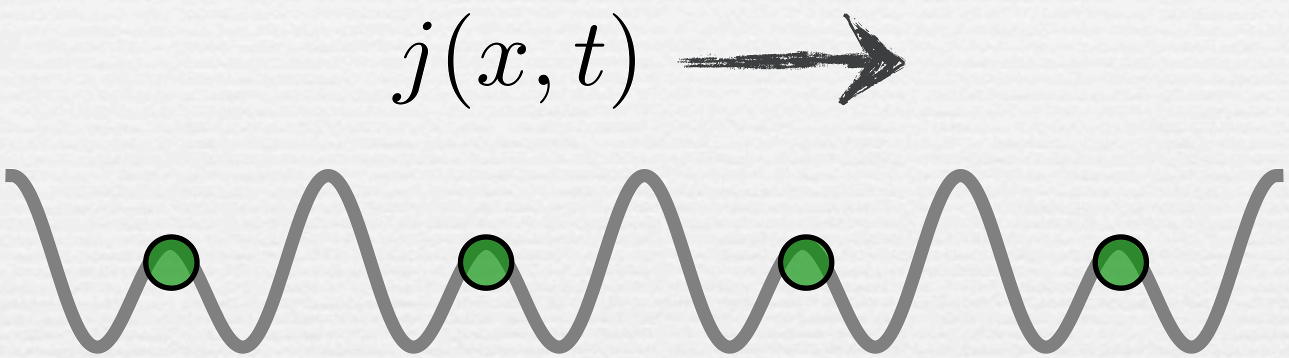


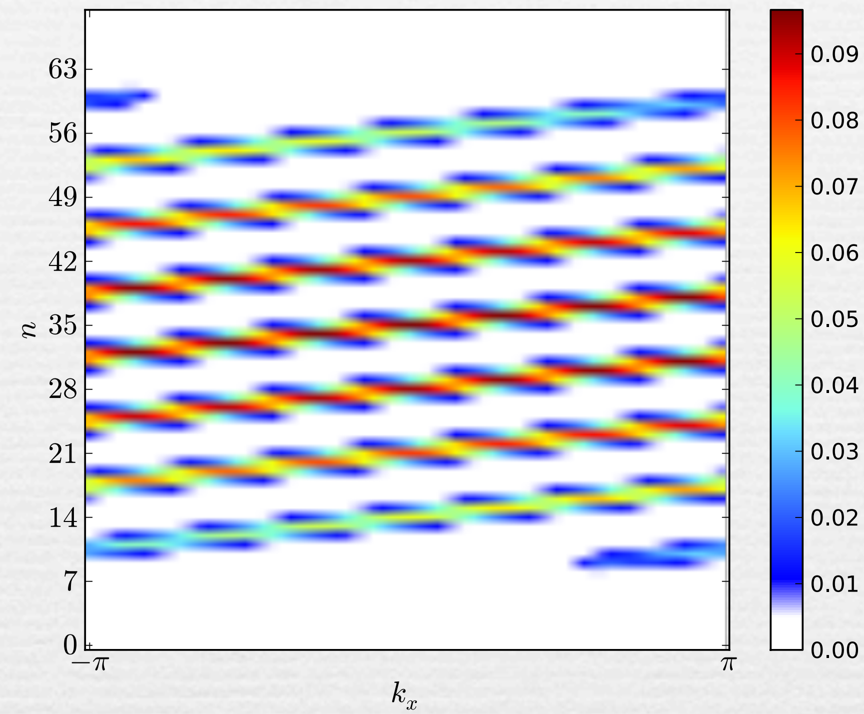


HTOF is also useful to detect FQHE state !

Summary

arXiv:1301.7435
PRL 110, 166802





Topological charge pumping is a common thread unifies many features of topological states

Guideline for design and detection of topological phases in cold atom systems

Summary

arXiv:1301.7435
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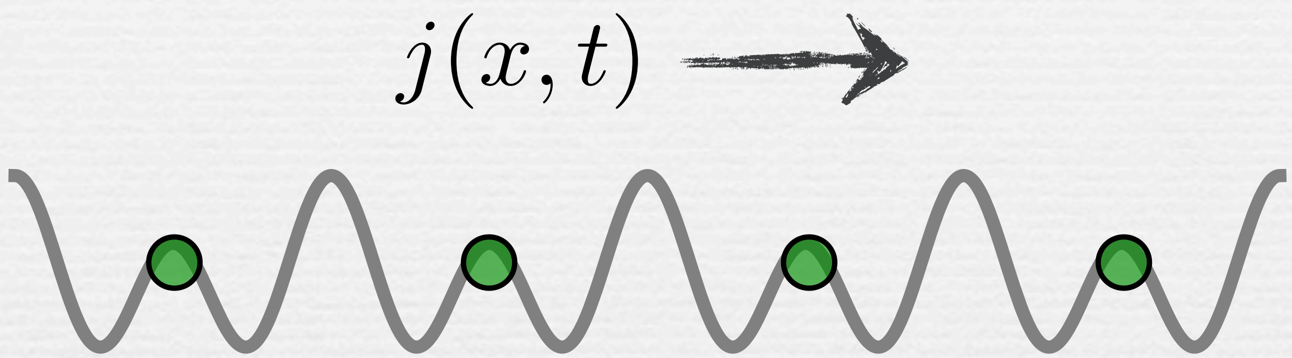

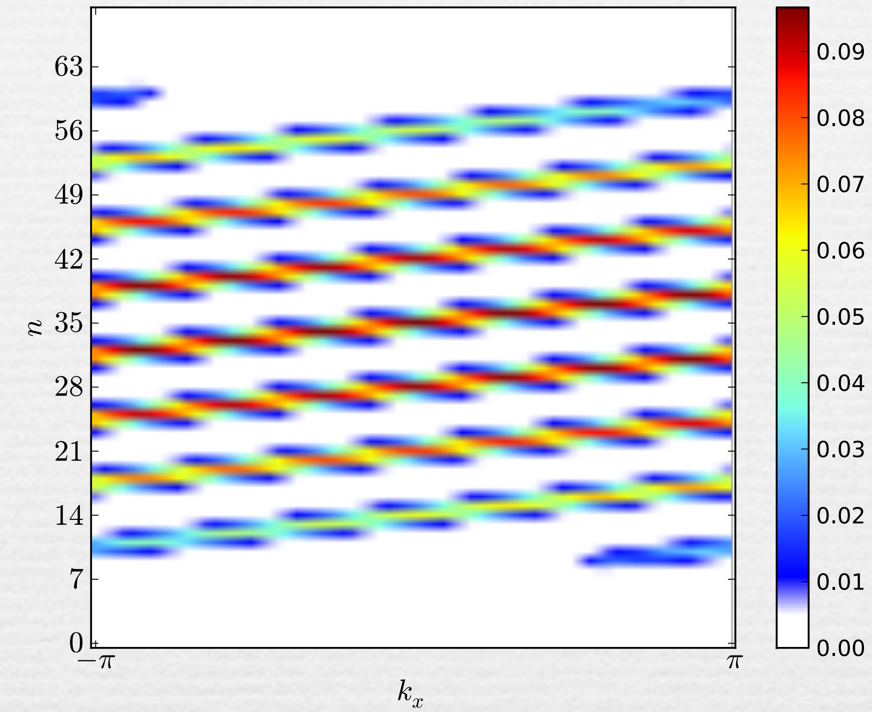

Topological charge pumping is a common thread unifies many features of topological states

Guideline for design and detection of topological phases in cold atom systems

You might try it in your lab !

Thank you!