

$$e^{\Delta\tau U(\hat{n}_{\mathbf{r}\uparrow} - \frac{1}{2})(\hat{n}_{\mathbf{r}\downarrow} - \frac{1}{2})} = \frac{e^{-\Delta\tau U/4}}{2} \sum_{s=\pm 1} e^{\alpha s(\hat{n}_{\mathbf{r}\uparrow} + \hat{n}_{\mathbf{r}\downarrow} - 1)}$$