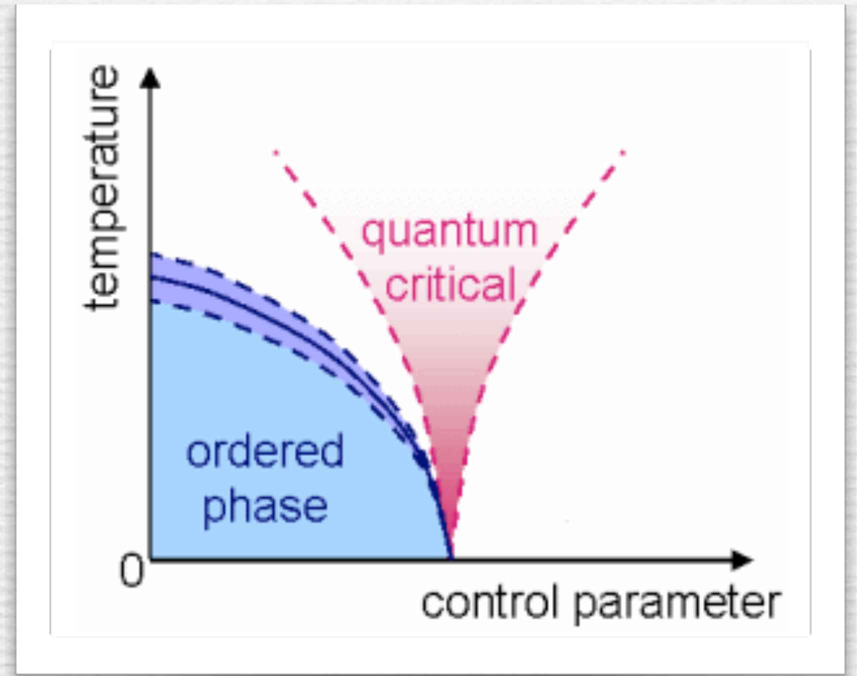
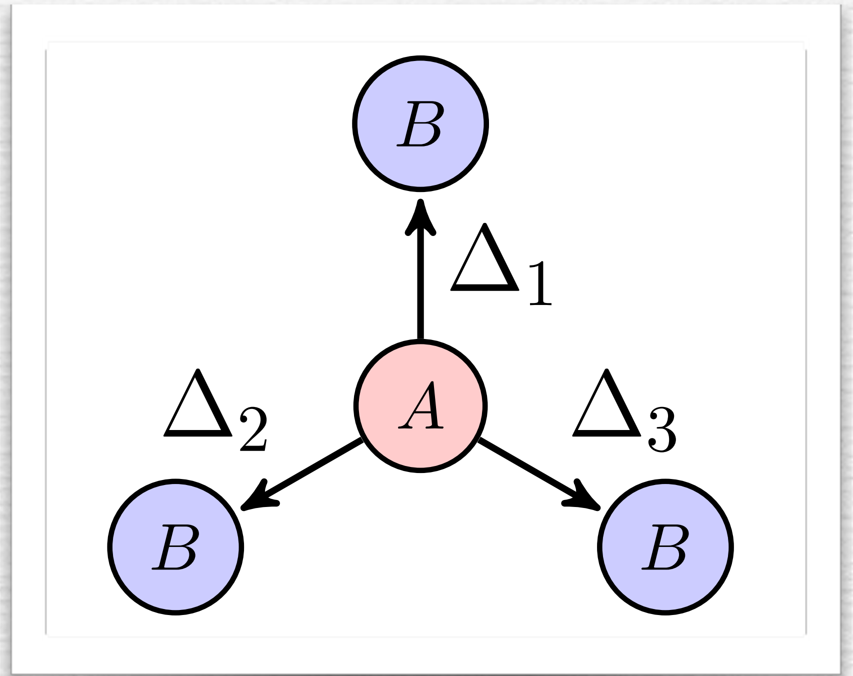
Spinless Fermions on a Honeycomb Lattice

From quantum criticality to topological superconductors





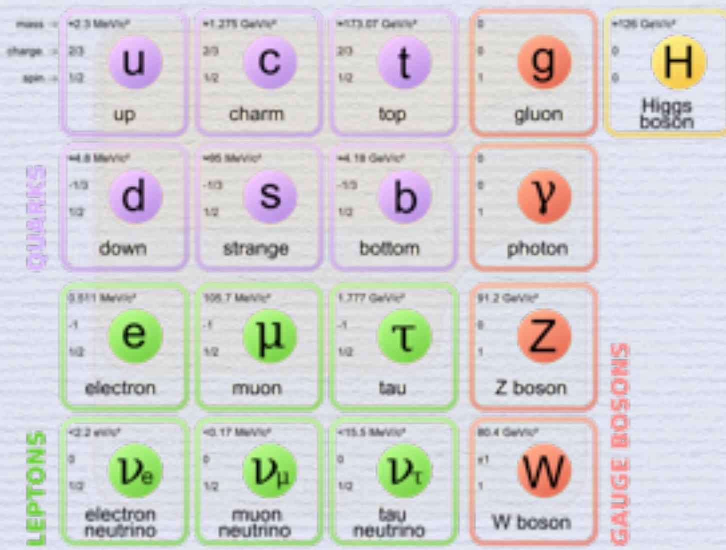
Lei Wang
ETH Zurich

Collaborators
Philippe Corboz
Matthias Troyer

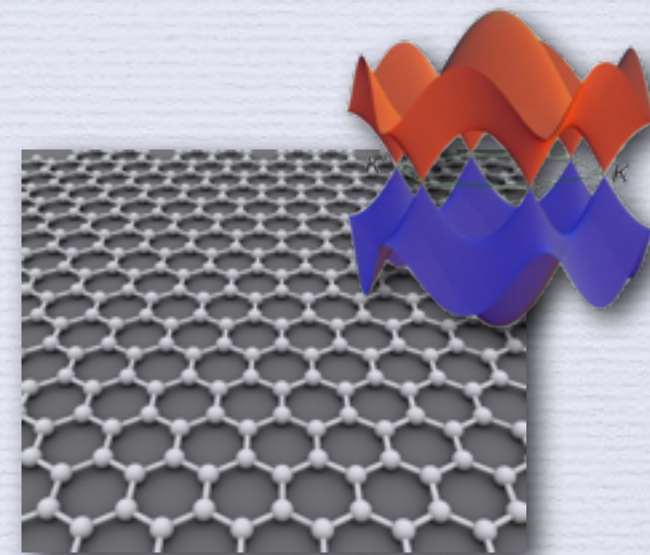
KITPC Program *"Precision Many-Body Physics of Strongly Correlated Quantum Matter"*

Dirac Fermions

Elementary Particles



Graphene

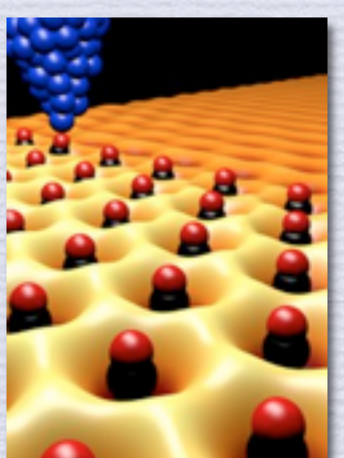


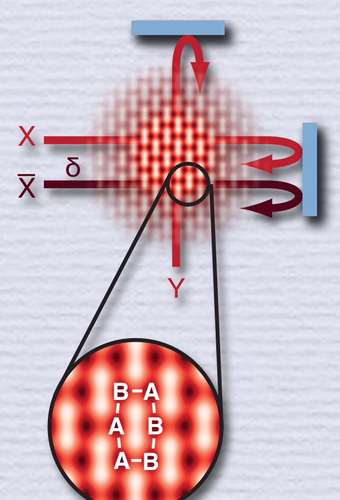
Novoselov et al, Zhang, et al, Nature, 2005

"Artificial graphene"

Gomes et al, Nature, 2012

Tarruell et al, Nature, 2012

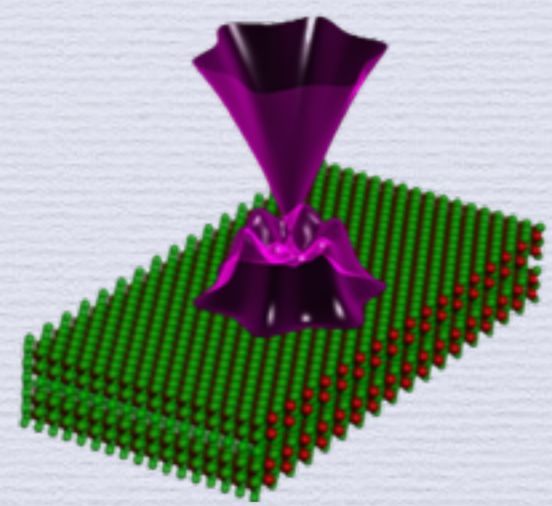




Many others...

Topological insulator

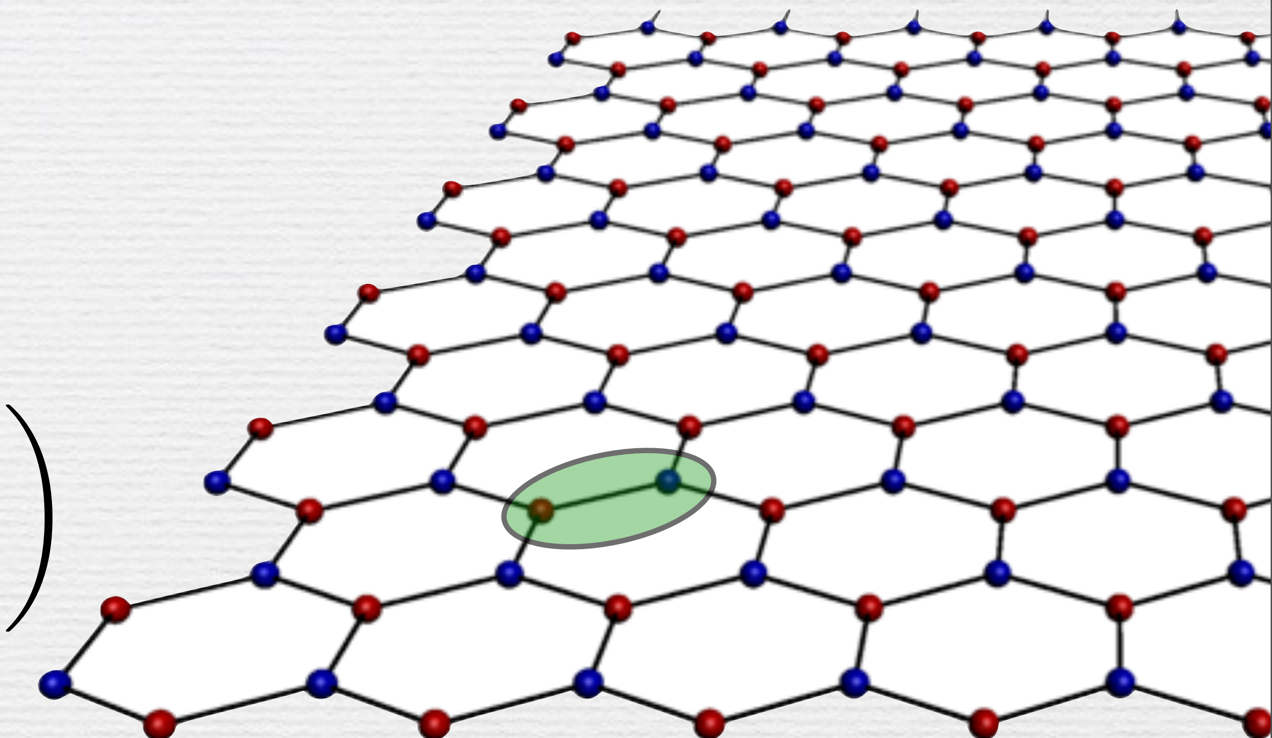
d-wave superconductor

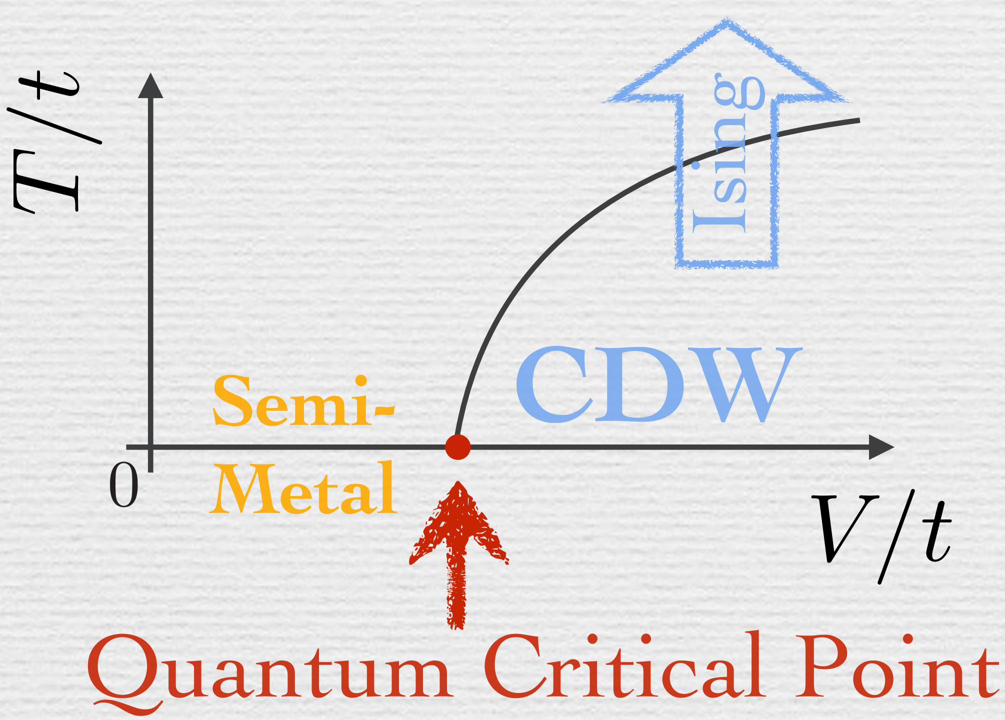


Spinless fermions on a honeycomb lattice

$$\hat{H}_0 = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{c}^\dagger_{\mathbf{i}} \hat{c}_{\mathbf{j}} + h.c.$$

$$\hat{H}_1 = V \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right)$$

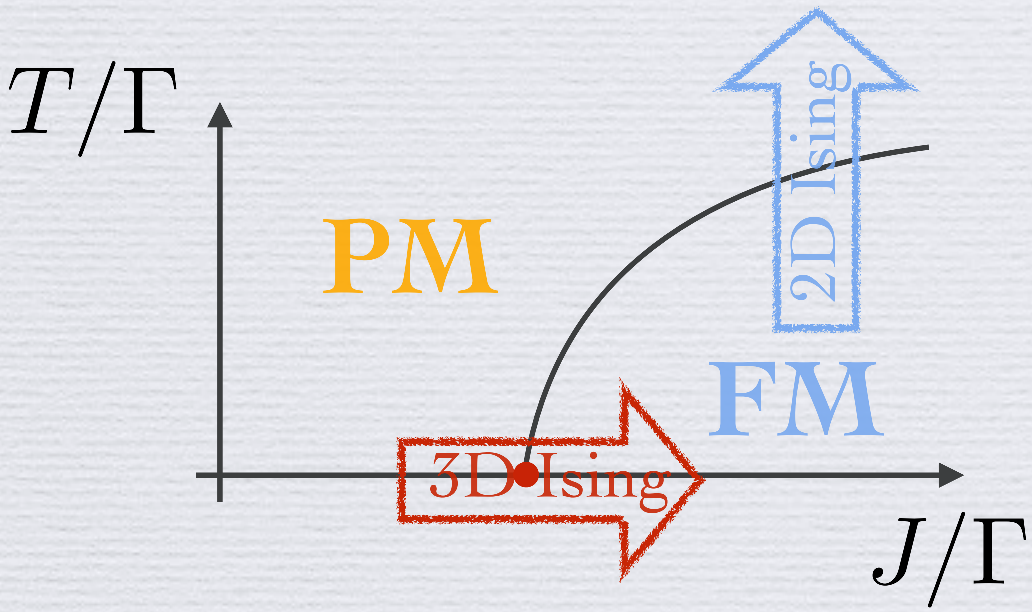




- Where is the QCP ?
- What is the universality class ?
- What are the critical exponents ?

Quantum Critical Points

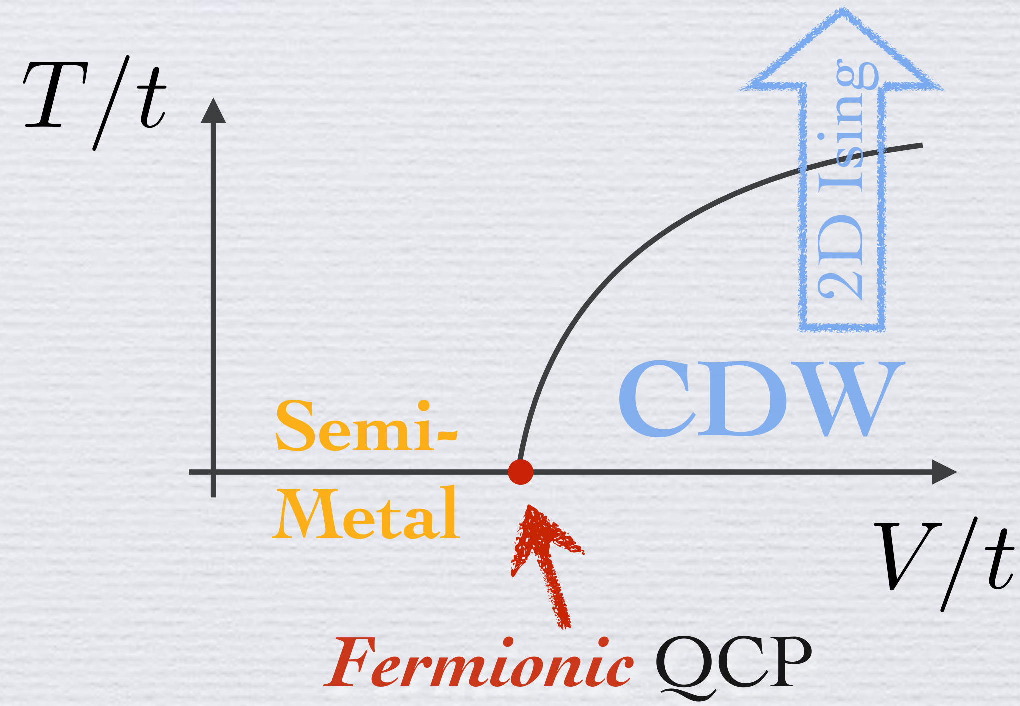
$$H_{\mathrm{TFIM}} = -J \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \sigma^z_{\mathbf{i}} \sigma^z_{\mathbf{j}} + \Gamma \sum_{\mathbf{i}} \sigma^x_{\mathbf{i}}$$



Scalar ϕ^4-theory

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$H = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(\hat{c}^\dagger_{\mathbf{i}} \hat{c}_{\mathbf{j}} + h.c. \right) + V \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right)$$



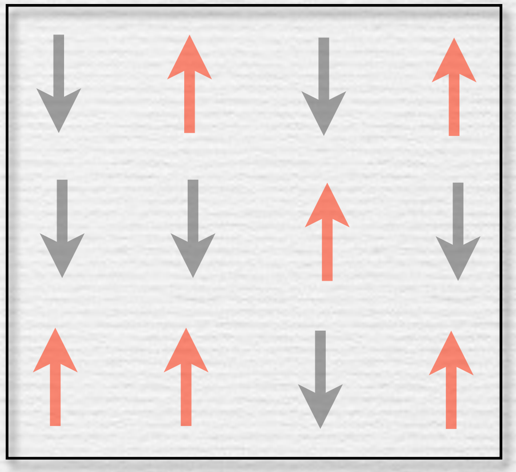
Gross-Neveu-Yukawa theory

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\Psi + g\phi\bar{\Psi}\sigma^z\Psi$$

Sign problem in auxiliary field QMC

Blankenbecler, et al, PRD, 1981

$$Z = \sum_{\mathcal{C}} w(\mathcal{C})$$



$\mathcal{C}$

- No sign problem: attractive Hubbard model with balanced filling

$$w(\mathcal{C}) = \det M_{\uparrow} \times \det M_{\downarrow}$$

$$= |\det M_{\uparrow}|^2 \geq 0$$

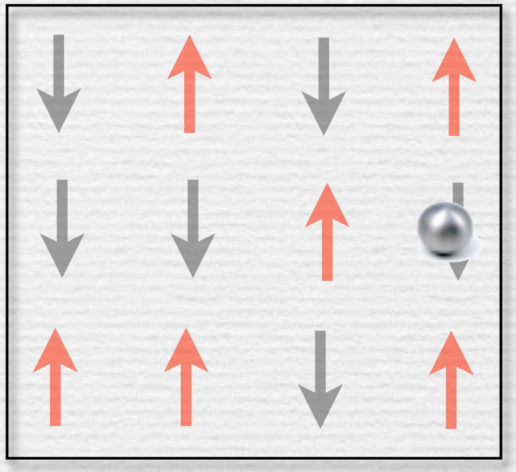
- How about spinless fermions ?

$$w(\mathcal{C}) = \det M$$

Sign problem in auxiliary field QMC

Blankenbecler, et al, PRD, 1981

$$Z = \sum_{\mathcal{C}} w(\mathcal{C})$$



$\mathcal{C}$

- No sign problem: attractive Hubbard model with balanced filling

$$w(\mathcal{C}) = \det M_{\uparrow} \times \det M_{\downarrow}$$

$$= |\det M_{\uparrow}|^2 \geq 0$$

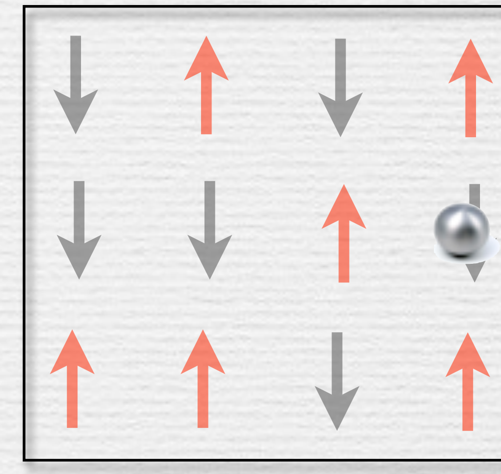
- How about spinless fermions ?

$$w(\mathcal{C}) = \det M$$

Sign problem in auxiliary field QMC

Blankenbecler, et al, PRD, 1981

$$Z = \sum_{\mathcal{C}} w(\mathcal{C})$$



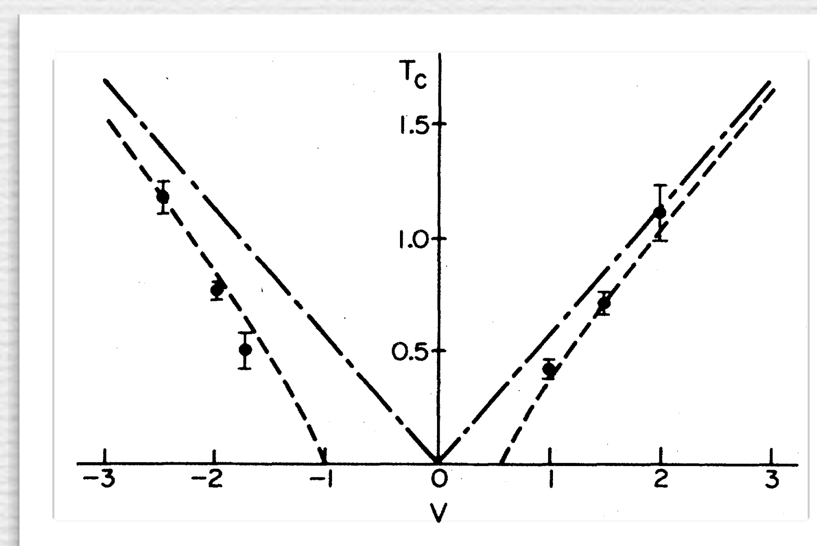
$\mathcal{C}$

- No sign problem: attractive Hubbard model with balanced filling

$$w(\mathcal{C}) = \det M_{\uparrow} \times \det M_{\downarrow}$$

$$= |\det M_{\uparrow}|^2 \geq 0$$

Scalapino, et al, PRB, 1984
Gubernatis, et al, PRB, 1985



up to 8×8 square lattice, T=0.3t

- How about spinless fermions ?

$$w(\mathcal{C}) = \det M$$

Determinant = Pfaffian^2

For skew-symmetric matrices   Huffman and Chandrasekharan, PRB, 2014

$$M = -M^T$$

$$\det M = (\mathrm{pf}\, M)^2 \geq 0$$ Muir, 1882

Named after J. F. Pfaff (*1765-1825*), a teacher of Gauss

$$\mathrm{pf}\begin{pmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{pmatrix} = af - be + cd$$

Johann Friedrich Pfaff

Is M skew-symmetric ?

No

$$w(\mathcal{C}) = \det(\mathbb{I} + B_{N_\tau} \dots B_2 B_1)$$

Yes

in **continuous-time** QMC ...

CTQMC

Rubtsov et al, PRB,2005

Gull et al, RMP, 2011

$$\hat{H}_0 = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{c}_{\mathbf{i}}^{\dagger} \hat{c}_{\mathbf{j}} + h.c.$$

$$\hat{H}_1 = V \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right)$$

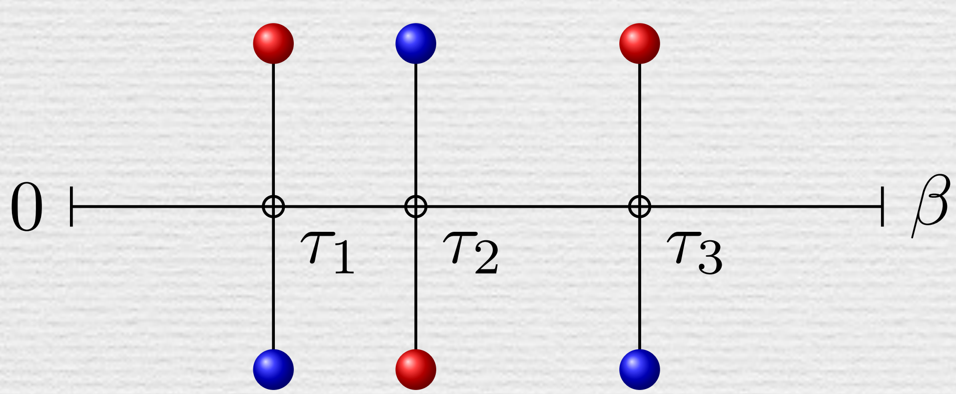
$$Z = \mathrm{Tr} \left[e^{-\beta \hat{H}_0} \mathcal{T} e^{-\int_0^\beta \hat{H}_1(\tau)} \right]$$

$$= \sum_{k=0}^{\infty} \int_0^{\beta} \mathrm{d}\tau_1 \ldots \int_{\tau_{k-1}}^{\beta} \mathrm{d}\tau_k \, (-1)^k \mathrm{Tr} \left[e^{-\beta \hat{H}_0} \hat{H}_1(\tau_k) \ldots \hat{H}_1(\tau_1) \right]$$

$$= \sum_{\mathcal{C}} w(\mathcal{C})$$

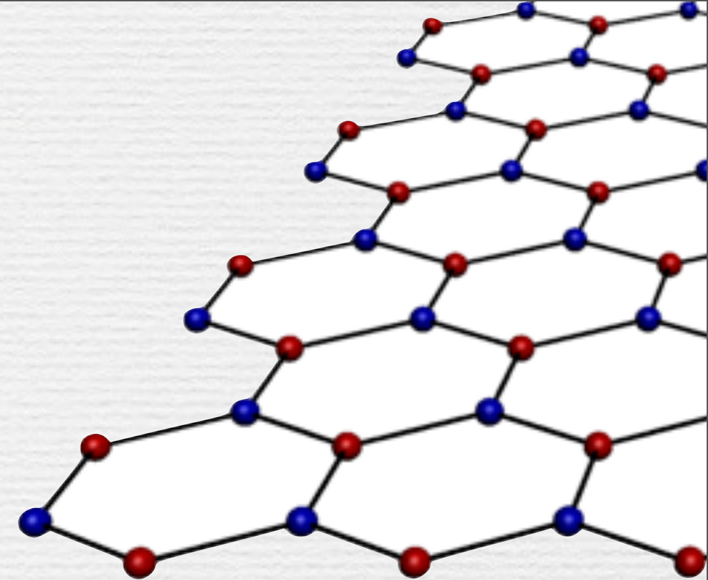
cf. Nikolay's tutorial talk

$$w(\mathcal{C}) = (-V)^k \det(G)$$



$$G = \begin{pmatrix} \text{Noninteracting} \\ \text{Green's functions} \end{pmatrix}_{2k \times 2k}$$

~~Sign Problem~~



Parity Matrix $D = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}_{2k\times 2k}$

Huffman and Chandrasekharan, PRB, 2014

$G^T = -DGD$ → $(GD)^T = -GD$

$$w(\mathcal{C}) = (-V)^k \det(G)$$

$$= (-V)^k \det(D) \det(GD)$$

$$= V^k \mathrm{pf}(GD)^2 \geq 0$$

*half-filling ensures diagonal element vanishes

Observables & Scaling Ansatz

± 1 for A(B) sublattice

$$M_2 = \frac{1}{N^2} \sum_{\mathbf{i},\mathbf{j}} \eta_{\mathbf{i}} \eta_{\mathbf{j}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right) \right\rangle$$

$$M_4 = \frac{1}{N^4} \sum_{\mathbf{i},\mathbf{j},\mathbf{k},\mathbf{l}} \eta_{\mathbf{i}} \eta_{\mathbf{j}} \eta_{\mathbf{k}} \eta_{\mathbf{l}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{k}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{l}} - \frac{1}{2} \right) \right\rangle$$

$2L^2$ up to 450 sites

Observables & Scaling Ansatz

± 1 for A(B) sublattice

$$M_2 = \frac{1}{N^2}\sum_{\mathbf{i},\mathbf{j}} \eta_{\mathbf{i}}\eta_{\mathbf{j}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{j}} - \frac{1}{2}\right)\right\rangle$$

$$M_4 = \frac{1}{N^4}\sum_{\mathbf{i},\mathbf{j},\mathbf{k},\mathbf{l}} \eta_{\mathbf{i}}\eta_{\mathbf{j}}\eta_{\mathbf{k}}\eta_{\mathbf{l}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{j}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{k}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{l}} - \frac{1}{2}\right)\right\rangle$$

$2L^2$ up to 450 sites

Scalings ansatz close to the QCP

$$M_2 = L^{-z-\eta}\mathcal{F}[L^{1/\nu}(V - V_c), L^z/\beta]$$

$$M_4 = L^{-2z-2\eta}\mathcal{G}[L^{1/\nu}(V - V_c), L^z/\beta]$$

Observables & Scaling Ansatz

± 1 for A(B) sublattice

$$M_2 = \frac{1}{N^2} \sum_{\mathbf{i},\mathbf{j}} \eta_{\mathbf{i}} \eta_{\mathbf{j}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right) \right\rangle$$

$$M_4 = \frac{1}{N^4} \sum_{\mathbf{i},\mathbf{j},\mathbf{k},\mathbf{l}} \eta_{\mathbf{i}} \eta_{\mathbf{j}} \eta_{\mathbf{k}} \eta_{\mathbf{l}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{j}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{k}} - \frac{1}{2} \right) \left(\hat{n}_{\mathbf{l}} - \frac{1}{2} \right) \right\rangle$$

$2L^2$ up to 450 sites

relativistic invariance

$z = 1$

Scalings ansatz close to the QCP

$$M_2 = L^{-z-\eta} \mathcal{F}[L^{1/\nu}(V - V_c), L^z/\beta]$$

$$M_4 = L^{-2z-2\eta} \mathcal{G}[L^{1/\nu}(V - V_c), L^z/\beta]$$

Observables & Scaling Ansatz

± 1 for A(B) sublattice

$$M_2 = \frac{1}{N^2}\sum_{\mathbf{i},\mathbf{j}} \eta_{\mathbf{i}}\eta_{\mathbf{j}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{j}} - \frac{1}{2}\right)\right\rangle$$

$$M_4 = \frac{1}{N^4}\sum_{\mathbf{i},\mathbf{j},\mathbf{k},\mathbf{l}} \eta_{\mathbf{i}}\eta_{\mathbf{j}}\eta_{\mathbf{k}}\eta_{\mathbf{l}} \left\langle \left(\hat{n}_{\mathbf{i}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{j}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{k}} - \frac{1}{2}\right)\left(\hat{n}_{\mathbf{l}} - \frac{1}{2}\right)\right\rangle$$

$2L^2$ up to 450 sites

relativistic invariance

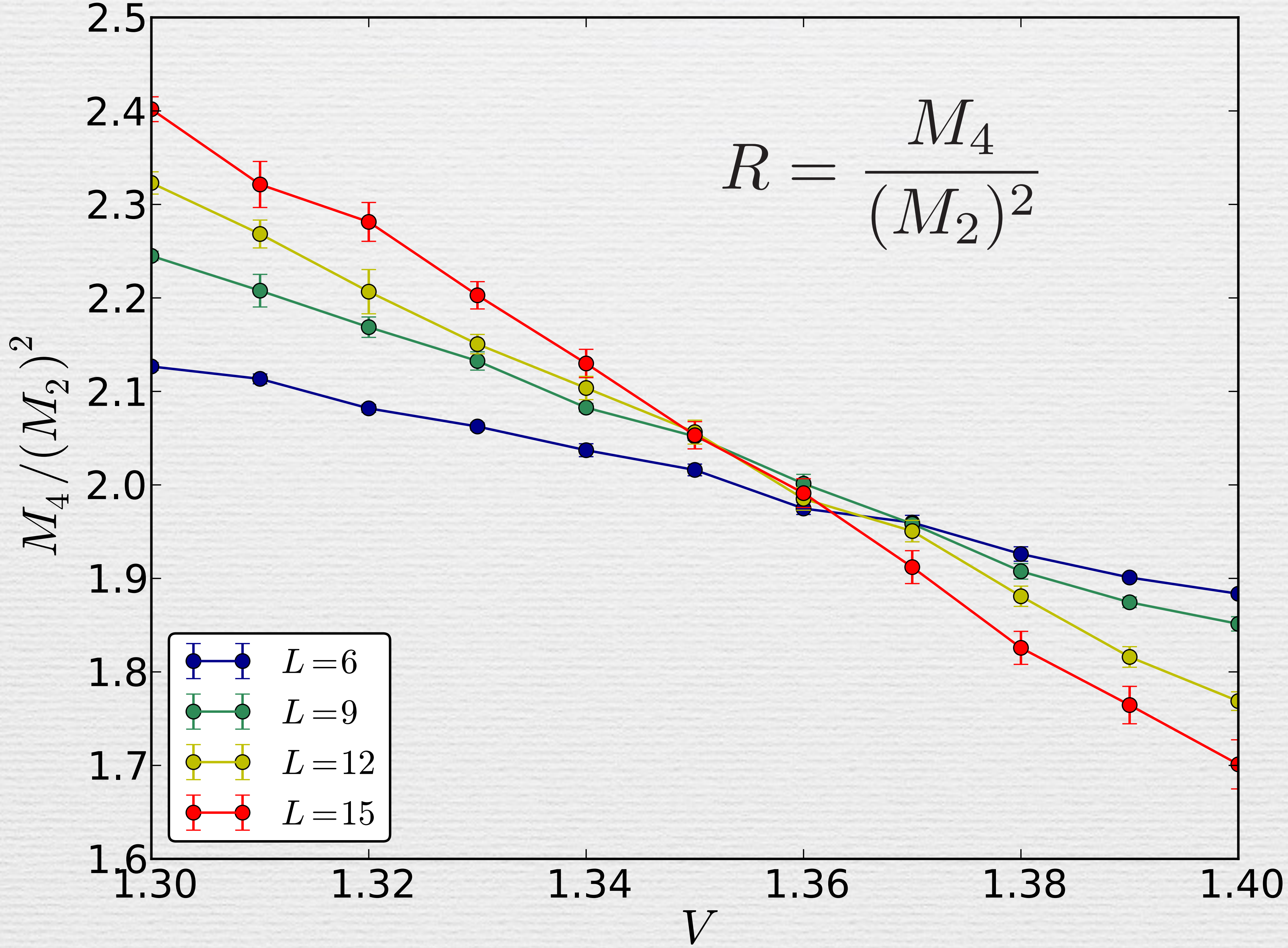
$z = 1$

Scalings ansatz close to the QCP

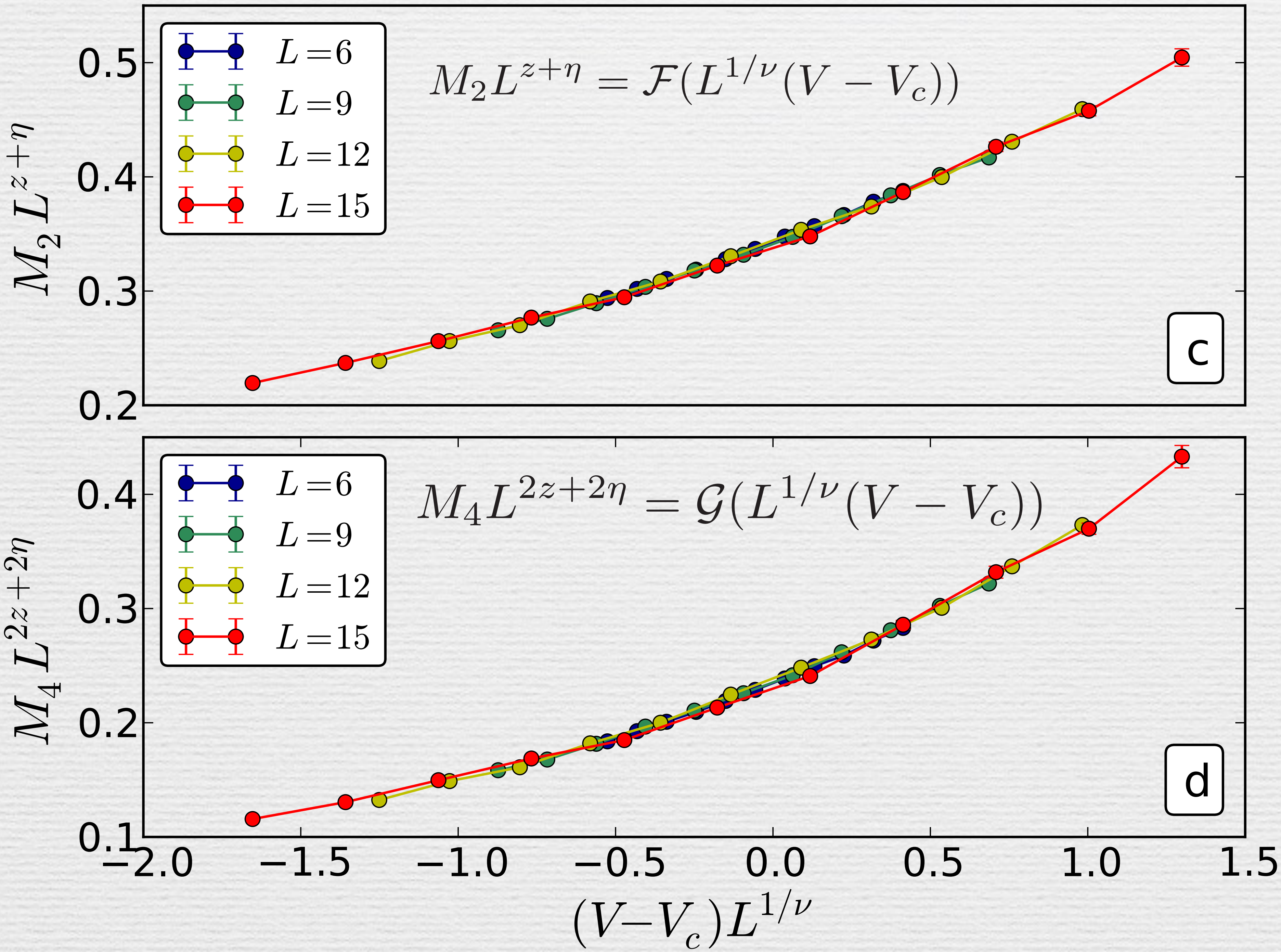
$$M_2 = L^{-z-\eta}\mathcal{F}[L^{1/\nu}(V - V_c), L^z/\beta]$$

$$M_4 = L^{-2z-2\eta}\mathcal{G}[L^{1/\nu}(V - V_c), L^z/\beta]$$

Binder Ratio



Data Collapse



$V_c/t = 1.356(1)$

$\nu = 0.80(3)$

$\eta = 0.302(7)$

* Errorbar from χ^2+1 analysis

Gross-Neveu-Yukawa Theory

- ε-expansion Rosenstein et al, PLB, 1993

$$\nu = 0.797 \qquad \eta = 0.502$$

- "Exact" renormalization group Rosa et al, PRL,2001 Höfling et al, PRB, 2002

$$\nu = 0.738 \sim 0.927 \quad \eta = 0.525 \sim 0.635$$

Honeycomb

$$\nu = 0.80(3)$$

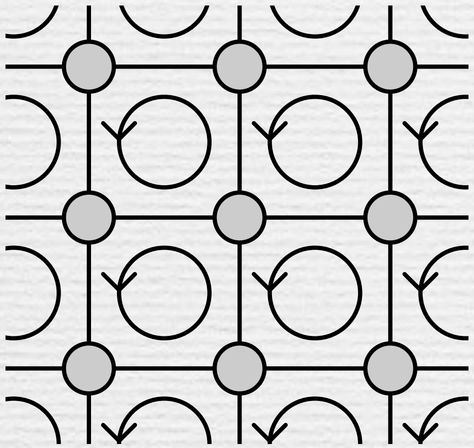
$$\eta = 0.302(7)$$

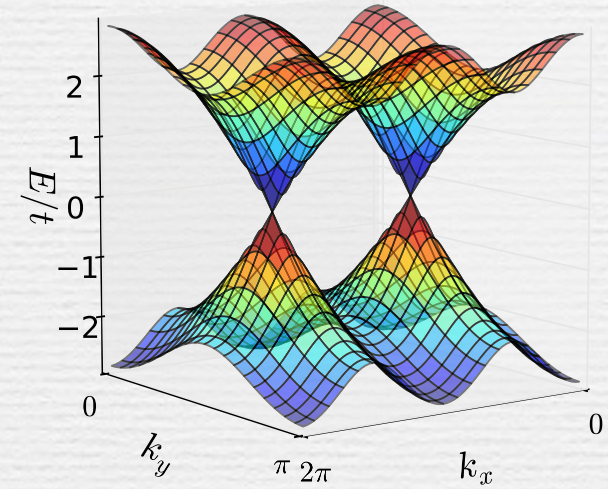
ν agrees

η does not

* Calculations based on 2-flavors of 2-component Dirac fermions with the same chirality

Check-I: π-flux square lattice





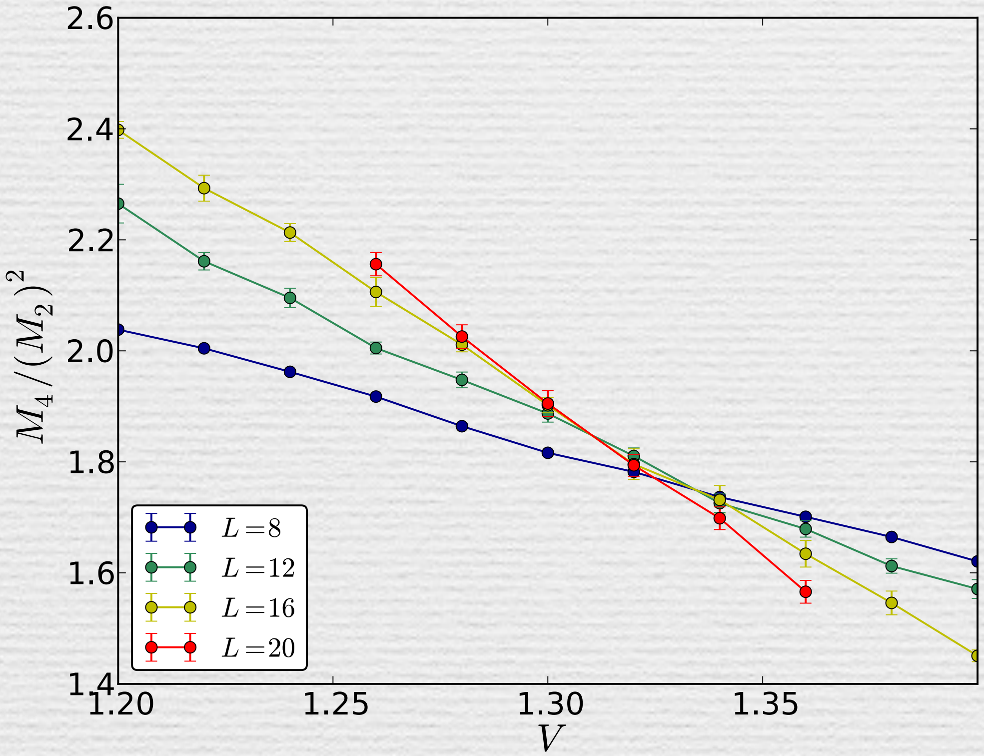
also features two Dirac points

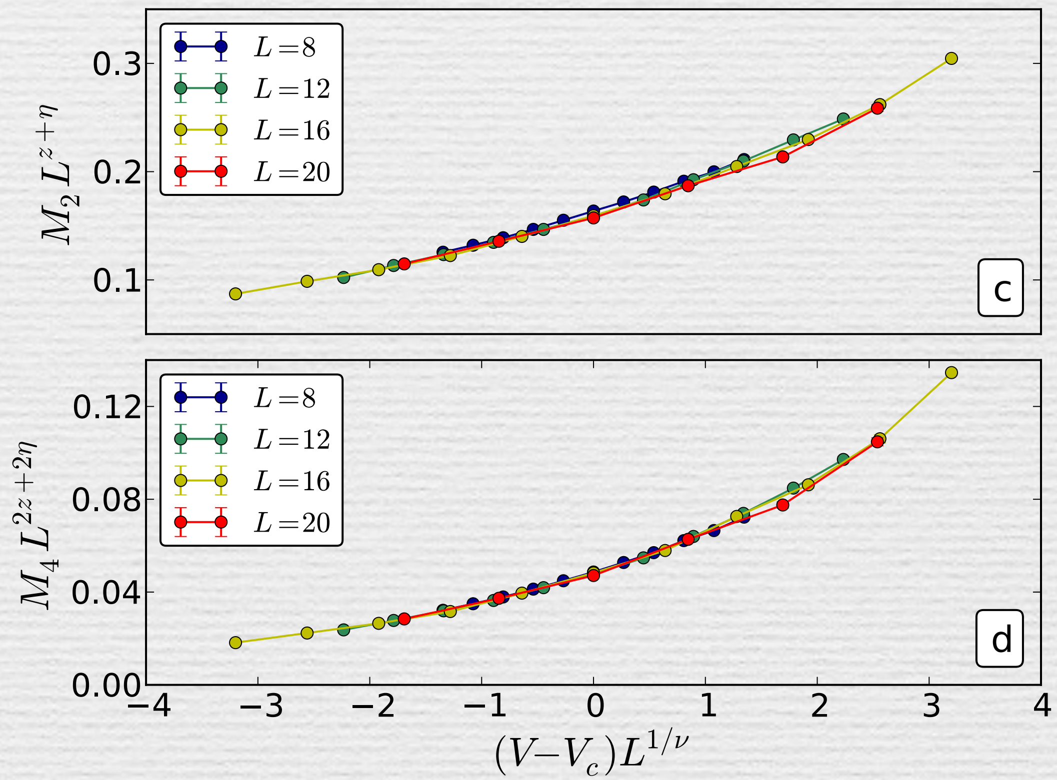
π-flux lattice

$$V_c/t = 1.304(2)$$

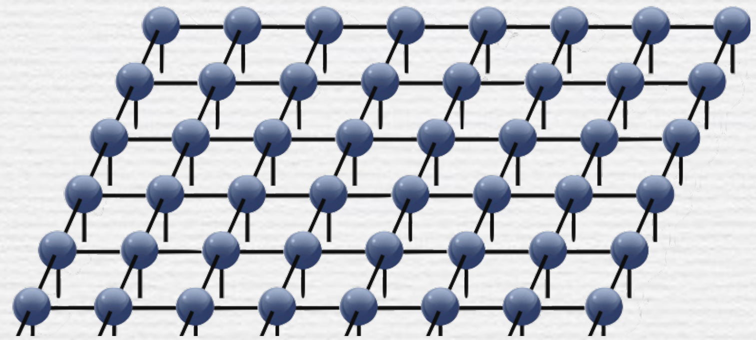
$$\nu = 0.80(6)$$

$$\eta = 0.318(8)$$

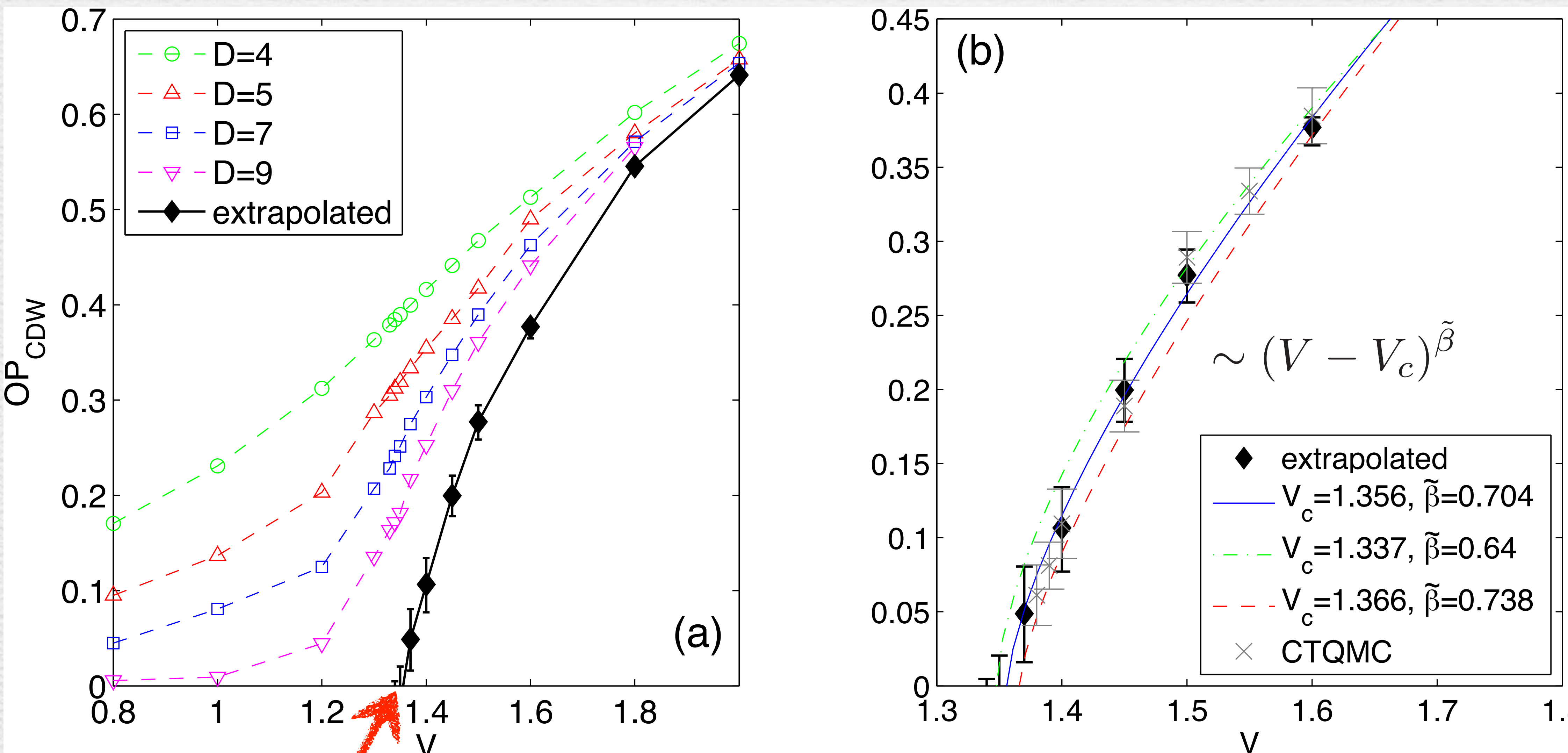




Check-II: iPEPS



Philippe Corboz
ETH→Amsterdam



$$V_c = 1.36(3)$$

CTQMC

$$\tilde{\beta} = \frac{\nu}{2}(z+\eta) = 0.52(3)$$

Summary-I

- Where is the QCP?
- What is the universality ?
- What are the critical exponents ?

CTQMC

$$V_c/t = 1.356(1)$$

$$\nu = 0.80(3)$$

$$\eta = 0.302(7)$$

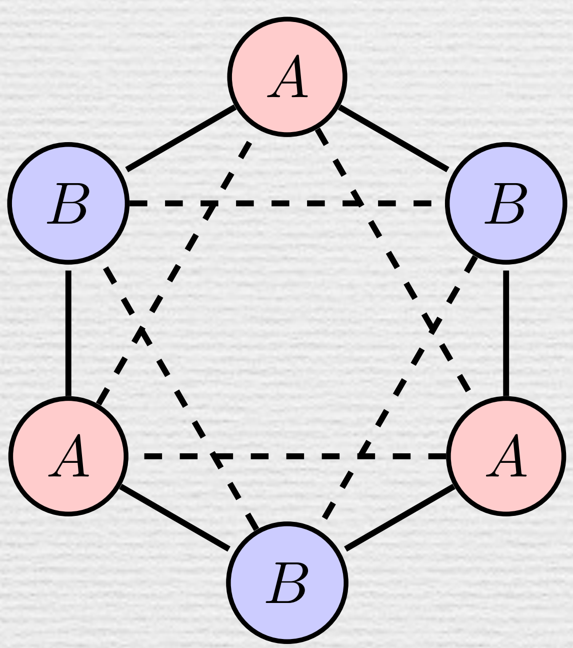
$$\tilde{\beta} = 0.52(3)$$

To resolve the discrepancies we need
bigger systems and more careful data analysis

Where do we go from here ?

Half-filled, nearest-neighbor repulsion

Next-nearest-neighbor repulsion

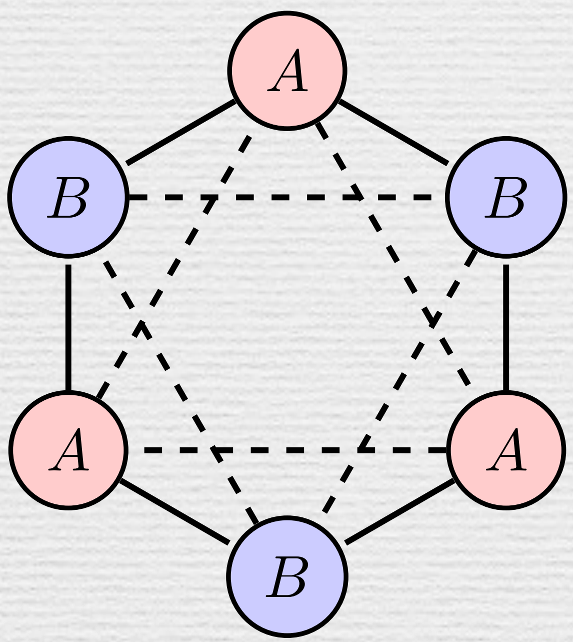


Doped attractive system

Where do we go from here ?

Half-filled, nearest-neighbor repulsion

Next-nearest-neighbor repulsion

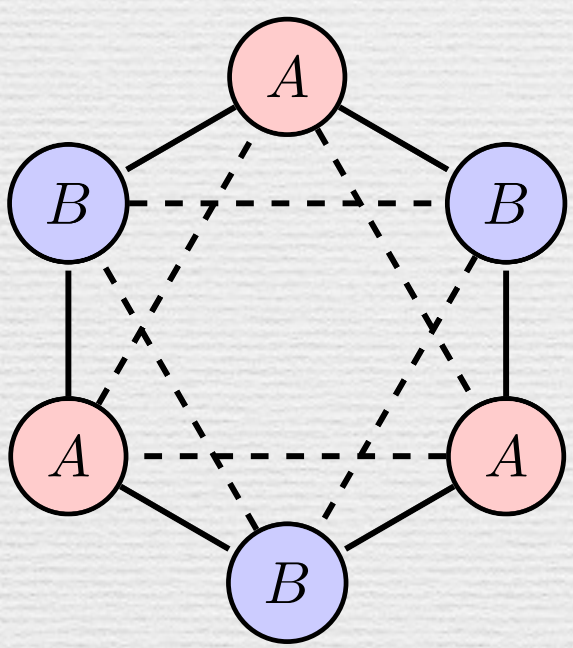


Doped attractive system

Where do we go from here ?

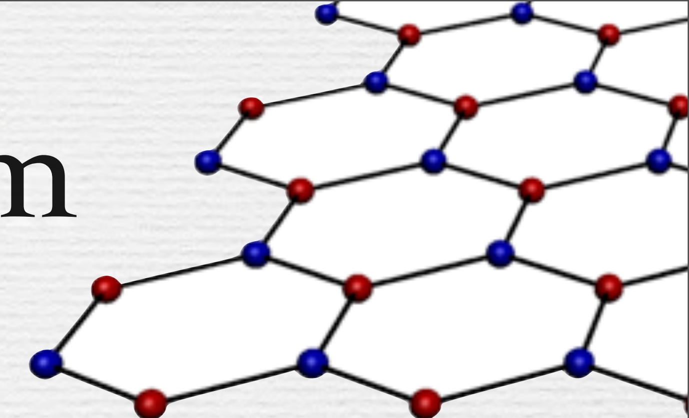
Half-filled, nearest-neighbor repulsion

Next-nearest-neighbor repulsion



Doped attractive system

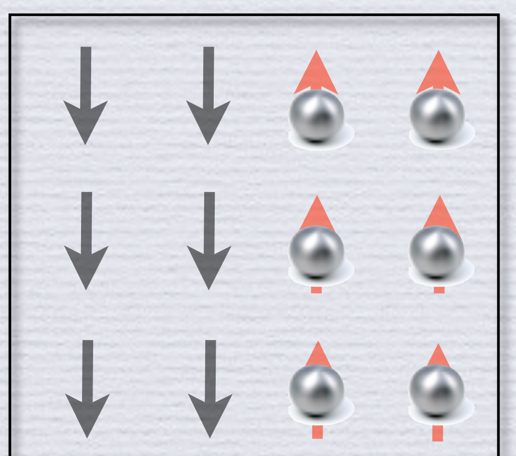
Doped attractive system



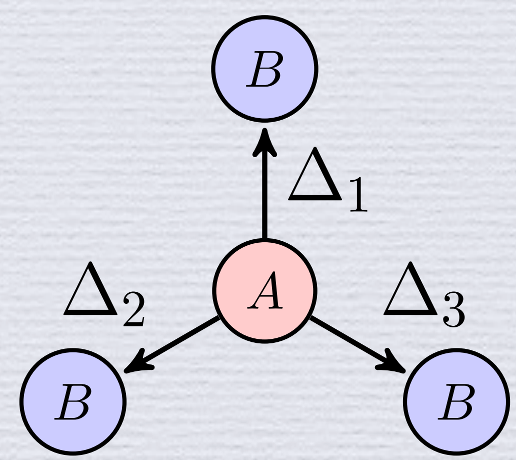
$V < 0$

$$\hat{H} = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(\hat{c}^\dagger_{\mathbf{i}} \hat{c}_{\mathbf{j}} + h.c. \right) + V \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{n}_{\mathbf{i}} \hat{n}_{\mathbf{j}} - \mu \sum_{\mathbf{i}} \hat{n}_{\mathbf{i}}$$

Strong coupling
"Phase separation"



Weak-coupling
Superconductivity

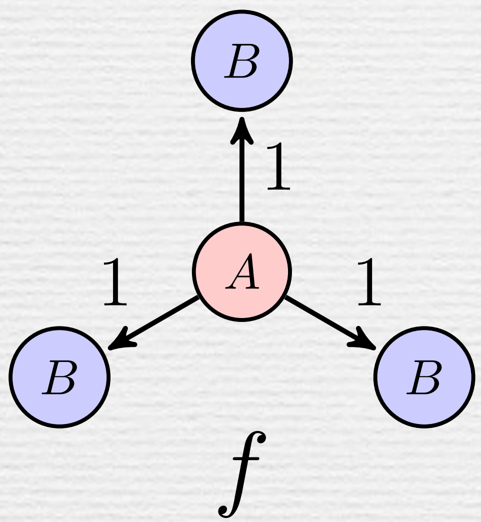


$$\Delta_\delta = \langle \hat{c}_{\mathbf{i} \in A} \hat{c}_{\mathbf{i}+\delta} \rangle$$

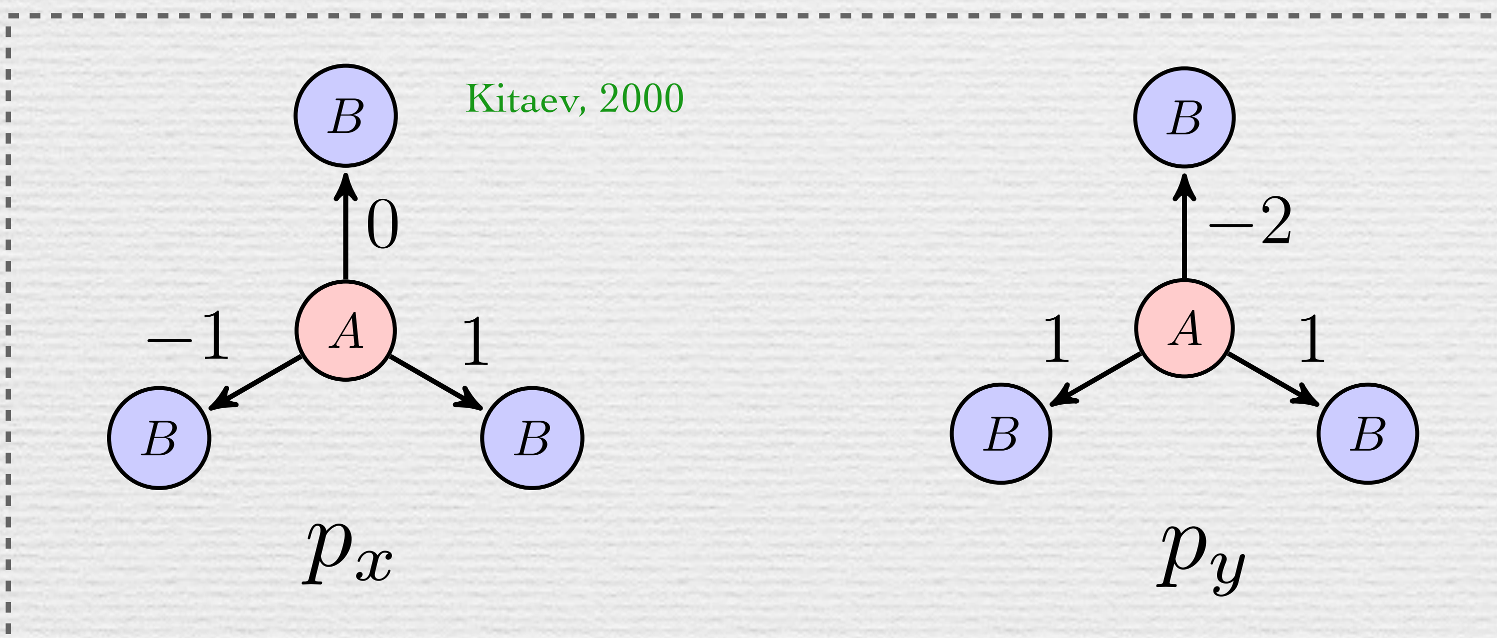
Pairing Symmetries

D_6	E	C_2	$2C_3$	$2C_6$	$3C_2'$	$3C_2''$
A_1	1	1	1	1	1	1
A_2	1	1	1	1	-1	-1
B_1	1	-1	1	-1	1	-1
B_2	1	-1	1	-1	-1	1
E_1	2	-2	-1	1	0	0
E_2	2	2	-1	-1	0	0

- 1-dim B_1 representation



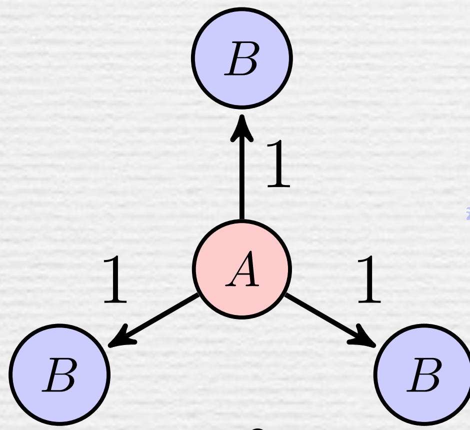
- 2-dim E_1 representation



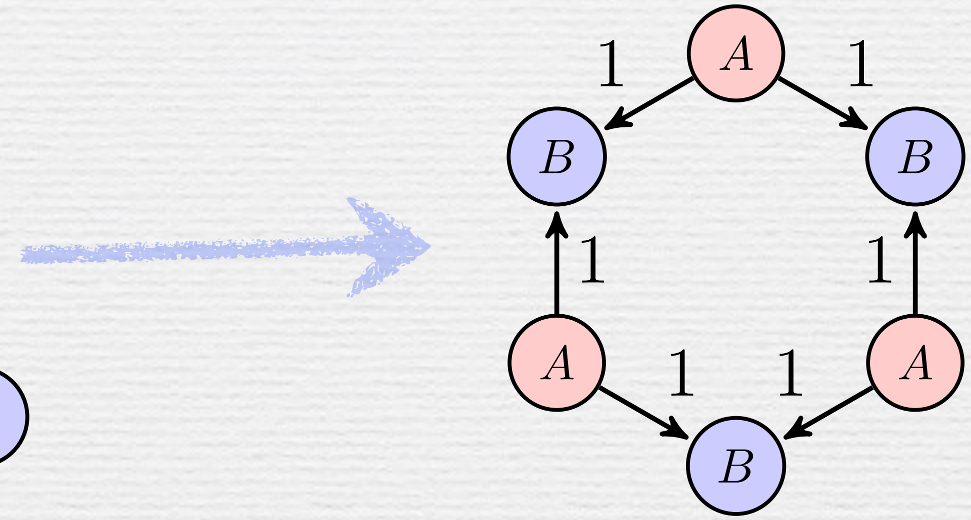
Pairing Symmetries

D_6	E	C_2	$2C_3$	$2C_6$	$3C_2'$	$3C_2''$
A_1	1	1	1	1	1	1
A_2	1	1	1	1	−1	−1
B_1	1	−1	1	−1	1	−1
B_2	1	−1	1	−1	−1	1
E_1	2	−2	−1	1	0	0
E_2	2	2	−1	−1	0	0

- 1-dim B_1 representation

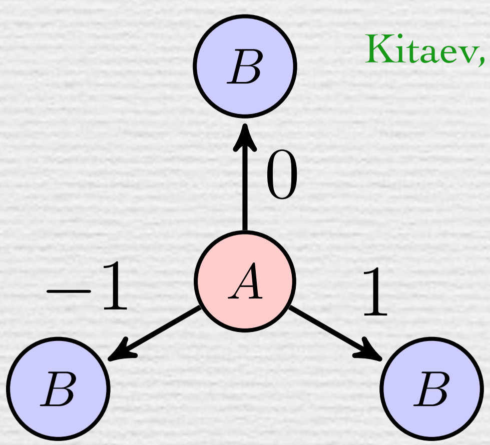


f

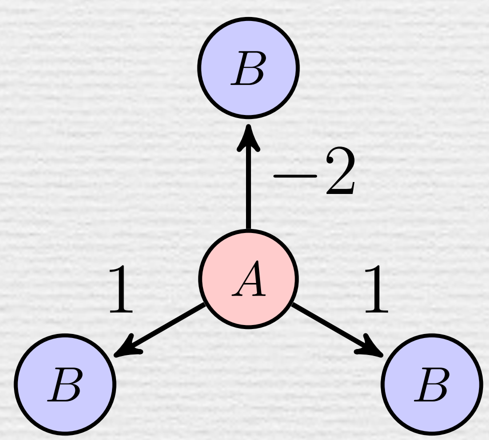


- 2-dim E_1 representation

Kitaev, 2000



p_x

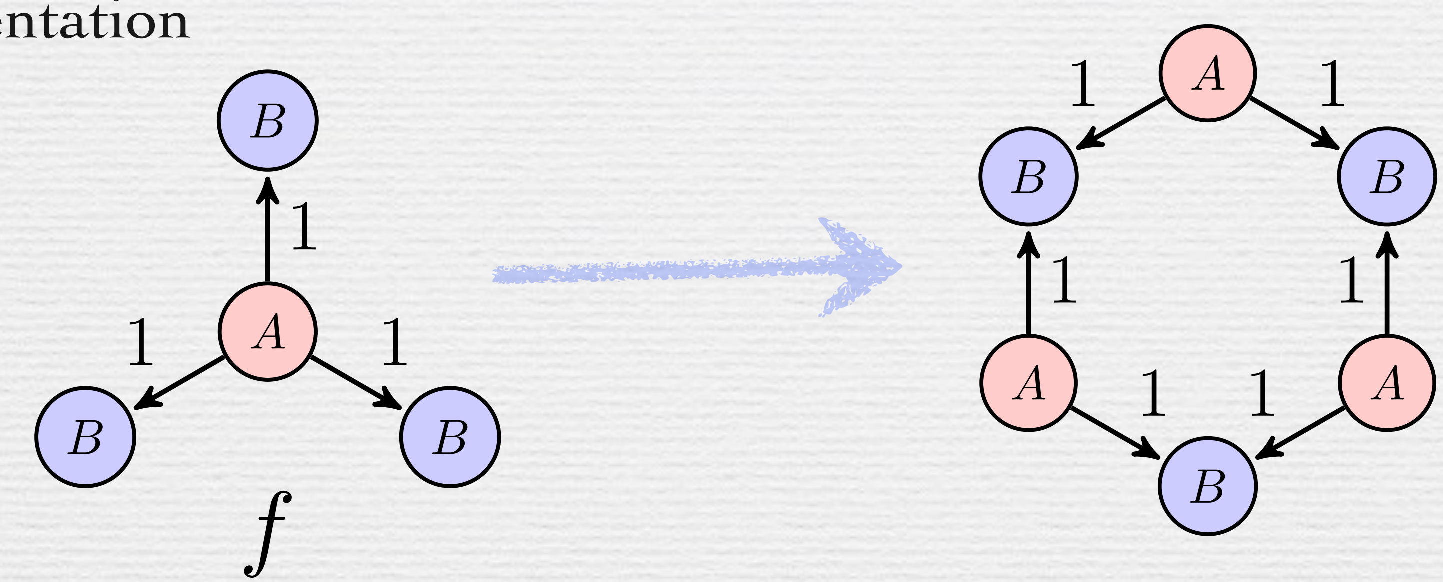


p_y

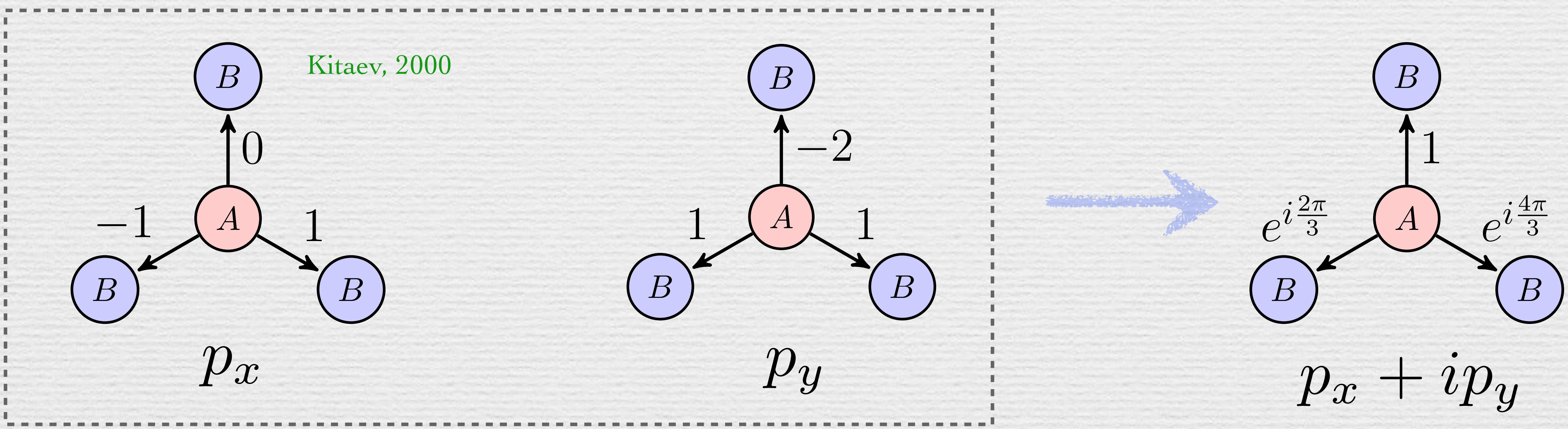
Pairing Symmetries

D_6	E	C_2	$2C_3$	$2C_6$	$3C_2'$	$3C_2''$
A_1	1	1	1	1	1	1
A_2	1	1	1	1	−1	−1
B_1	1	−1	1	−1	1	−1
B_2	1	−1	1	−1	−1	1
E_1	2	−2	−1	1	0	0
E_2	2	2	−1	−1	0	0

- 1-dim B_1 representation



- 2-dim E_1 representation



Read and Green, PRB, 2000

Pairing Susceptibilities

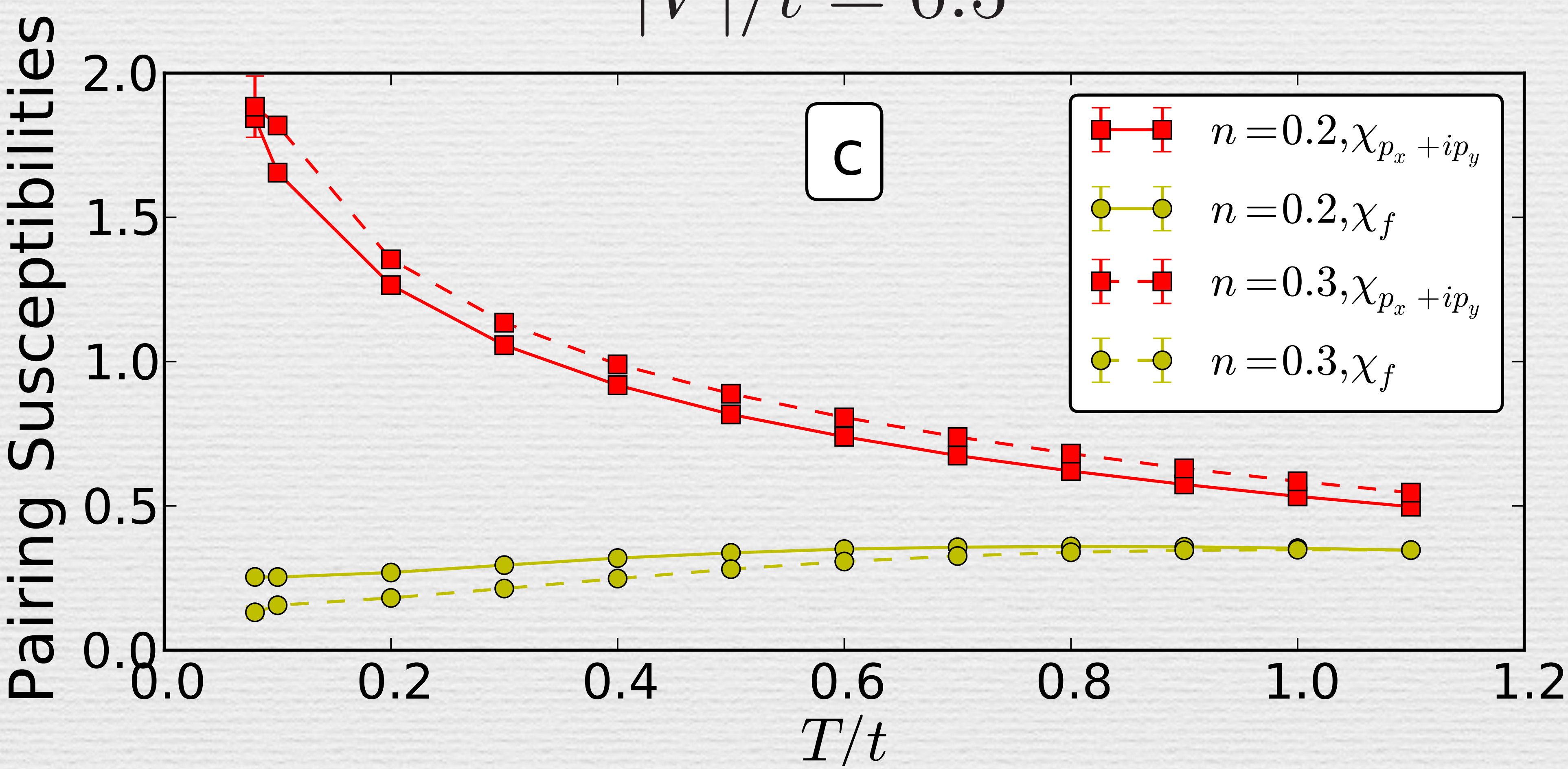
$$\chi_\Gamma = \int_0^\beta d\tau \langle \hat{\Delta}_\Gamma(\tau)\hat{\Delta}_\Gamma^\dagger \rangle$$

$$\hat{\Delta}_\Gamma(\tau) = \frac{1}{L^2}\sum_{\mathbf{i}\in A}\sum_{\delta} \mathcal{F}_\Gamma^\delta \hat{c}_{\mathbf{i}}(\tau)\hat{c}_{\mathbf{i}+\delta}(\tau)$$

$$\mathcal{F}_f = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathcal{F}_{p_x+ip_y} = \begin{pmatrix} 1 \\ e^{i2\pi/3} \\ e^{i4\pi/3} \end{pmatrix}$$

$|V|/t = 0.5$



BdG Calculation

$$H_{\mathrm{BdG}} =$$

$$\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}^{\dagger} \left(\begin{array}{cc|cc} -\mu & z_{\mathbf{k}} & 0 & \Delta_{\mathbf{k}} \\ z^{*}_{\mathbf{k}} & -\mu & -\Delta_{-\mathbf{k}} & 0 \\ \hline 0 & -\Delta^{*}_{-\mathbf{k}} & \mu & -z^{*}_{-\mathbf{k}} \\ \Delta^{*}_{\mathbf{k}} & 0 & -z_{-\mathbf{k}} & \mu \end{array}\right) \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}$$

BdG Calculation

$$-t(1 + e^{-i\mathbf{k}\mathbf{a}_2} + e^{i\mathbf{k}(\mathbf{a}_1 - \mathbf{a}_2)})$$

$$H_{\mathrm{BdG}} =$$

$$\frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}^{\dagger} \begin{pmatrix} -\mu & z_{\mathbf{k}} & 0 & \Delta_{\mathbf{k}} \\ z^*_{\mathbf{k}} & -\mu & -\Delta_{-\mathbf{k}} & 0 \\ 0 & -\Delta^*_{-\mathbf{k}} & \mu & -z^*_{-\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & 0 & -z_{-\mathbf{k}} & \mu \end{pmatrix} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}$$

BdG Calculation

$$-t(1 + e^{-i\mathbf{k}\mathbf{a}_2} + e^{i\mathbf{k}(\mathbf{a}_1 - \mathbf{a}_2)})$$

$$H_{\mathrm{BdG}} = \frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^\dagger_{-\mathbf{k}A} \\ \hat{c}^\dagger_{-\mathbf{k}B} \end{pmatrix}^\dagger \begin{pmatrix} -\mu & z_{\mathbf{k}} & 0 & \Delta_{\mathbf{k}} \\ z^*_{\mathbf{k}} & -\mu & -\Delta_{-\mathbf{k}} & 0 \\ 0 & -\Delta^*_{-\mathbf{k}} & \mu & -z^*_{-\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & 0 & -z_{-\mathbf{k}} & \mu \end{pmatrix} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^\dagger_{-\mathbf{k}A} \\ \hat{c}^\dagger_{-\mathbf{k}B} \end{pmatrix}$$

$$-V(\Delta_1 + \Delta_2 e^{-i\mathbf{k}\mathbf{a}_2} + \Delta_3 e^{i\mathbf{k}(\mathbf{a}_1 - \mathbf{a}_2)})$$

BdG Calculation

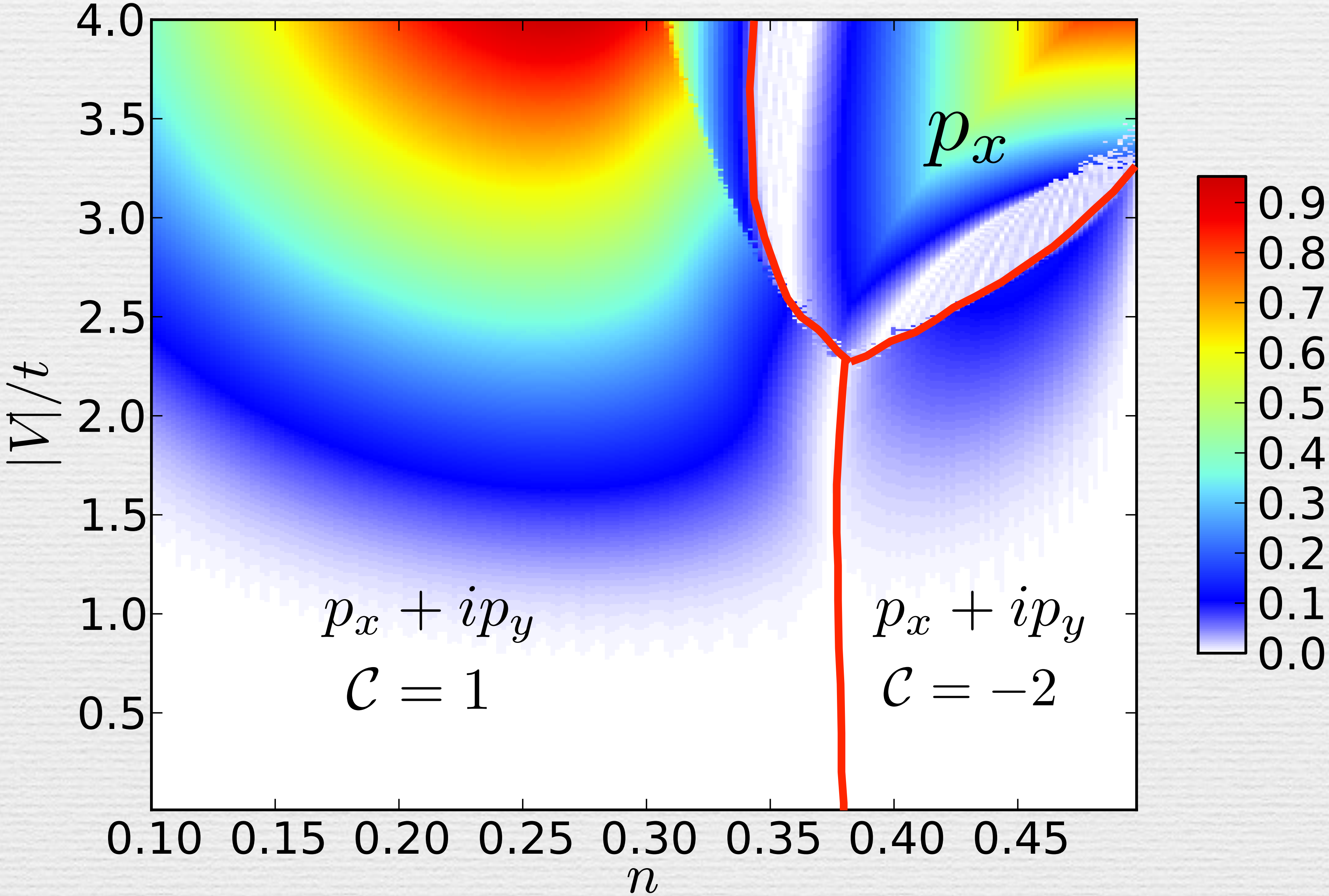
$$-t(1 + e^{-i\mathbf{k}\mathbf{a}_2} + e^{i\mathbf{k}(\mathbf{a}_1 - \mathbf{a}_2)})$$

$$H_{\mathrm{BdG}} = \frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}^{\dagger} \left(\begin{array}{cc|cc} -\mu & z_{\mathbf{k}} & 0 & \Delta_{\mathbf{k}} \\ z^*_{\mathbf{k}} & -\mu & -\Delta_{-\mathbf{k}} & 0 \\ \hline 0 & -\Delta^*_{-\mathbf{k}} & \mu & -z^*_{-\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & 0 & -z_{-\mathbf{k}} & \mu \end{array}\right) \begin{pmatrix} \hat{c}_{\mathbf{k}A} \\ \hat{c}_{\mathbf{k}B} \\ \hat{c}^{\dagger}_{-\mathbf{k}A} \\ \hat{c}^{\dagger}_{-\mathbf{k}B} \end{pmatrix}$$

$$-V(\Delta_1 + \Delta_2 e^{-i\mathbf{k}\mathbf{a}_2} + \Delta_3 e^{i\mathbf{k}(\mathbf{a}_1 - \mathbf{a}_2)})$$

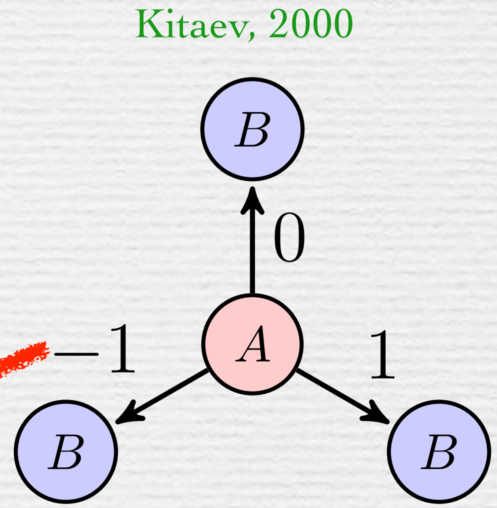
Start with random guesses, iterate until convergence

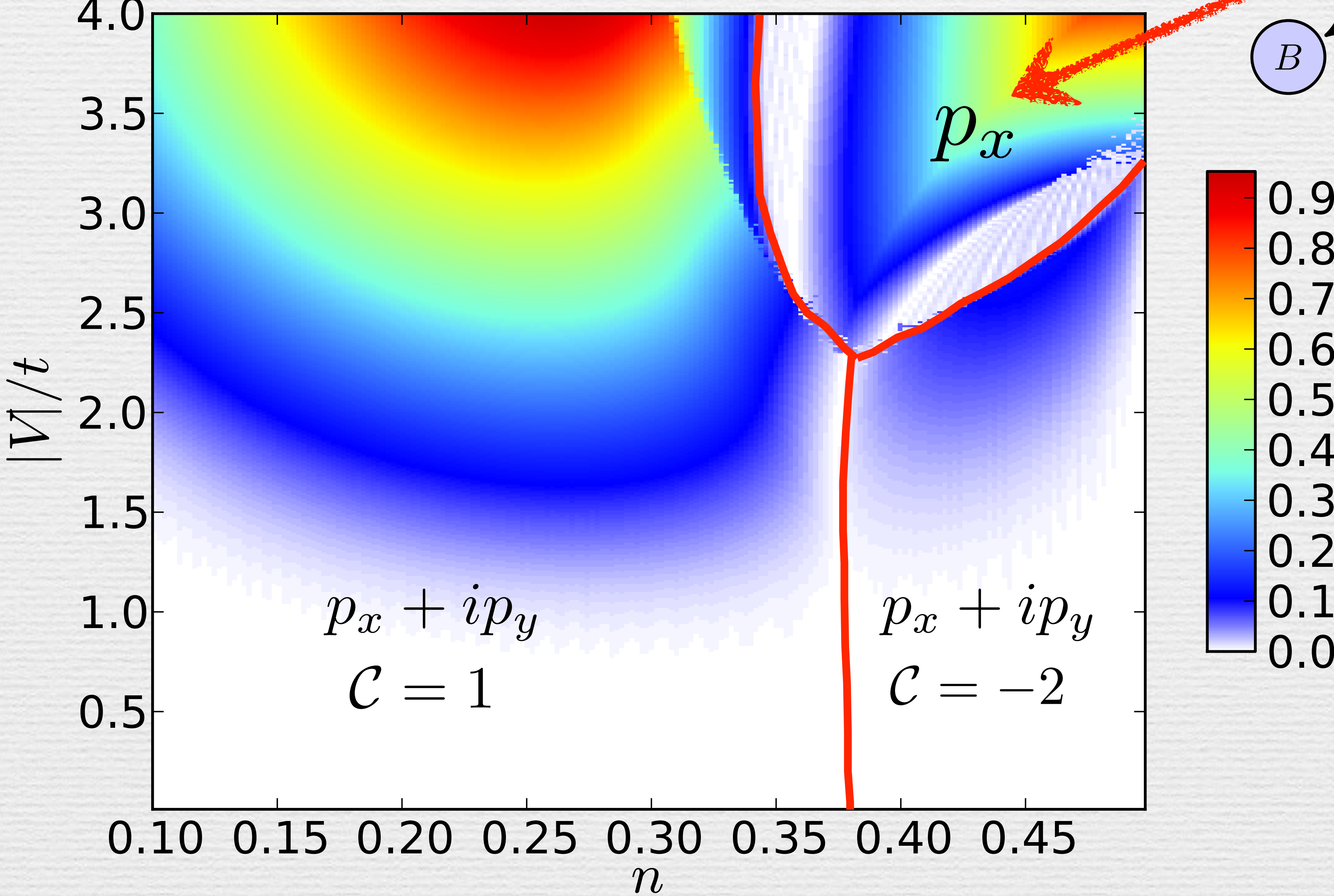
BdG phase diagram



*color indicates size of superconducting gap

BdG phase diagram

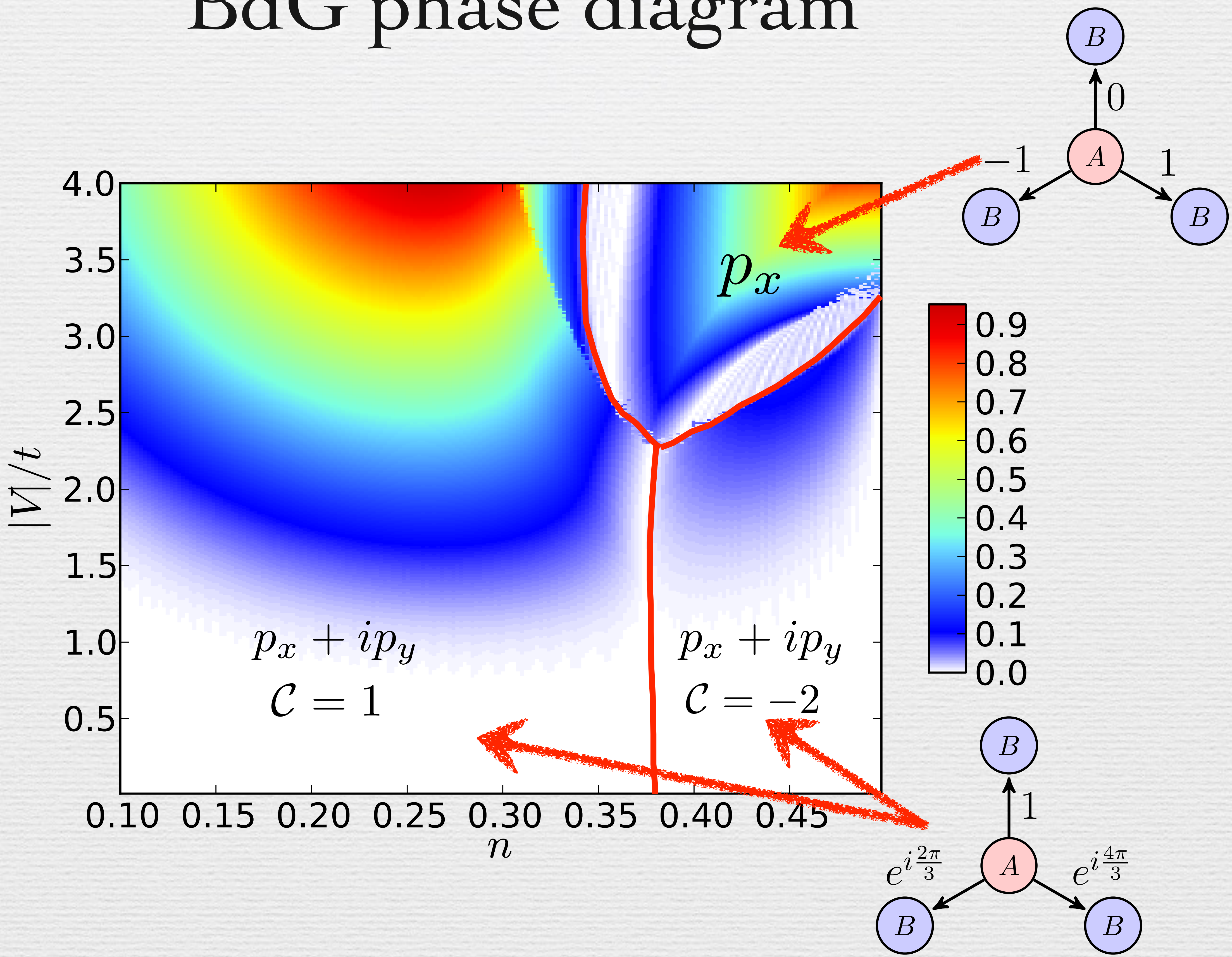




*color indicates size of superconducting gap

BdG phase diagram

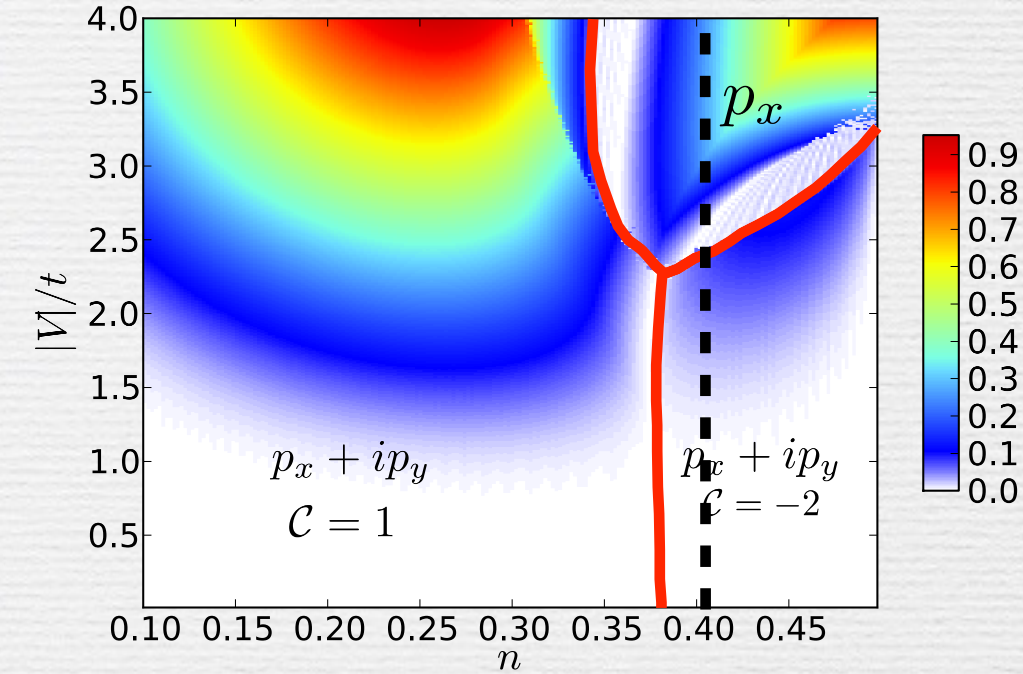
Kitaev, 2000



Read and Green, PRB, 2000

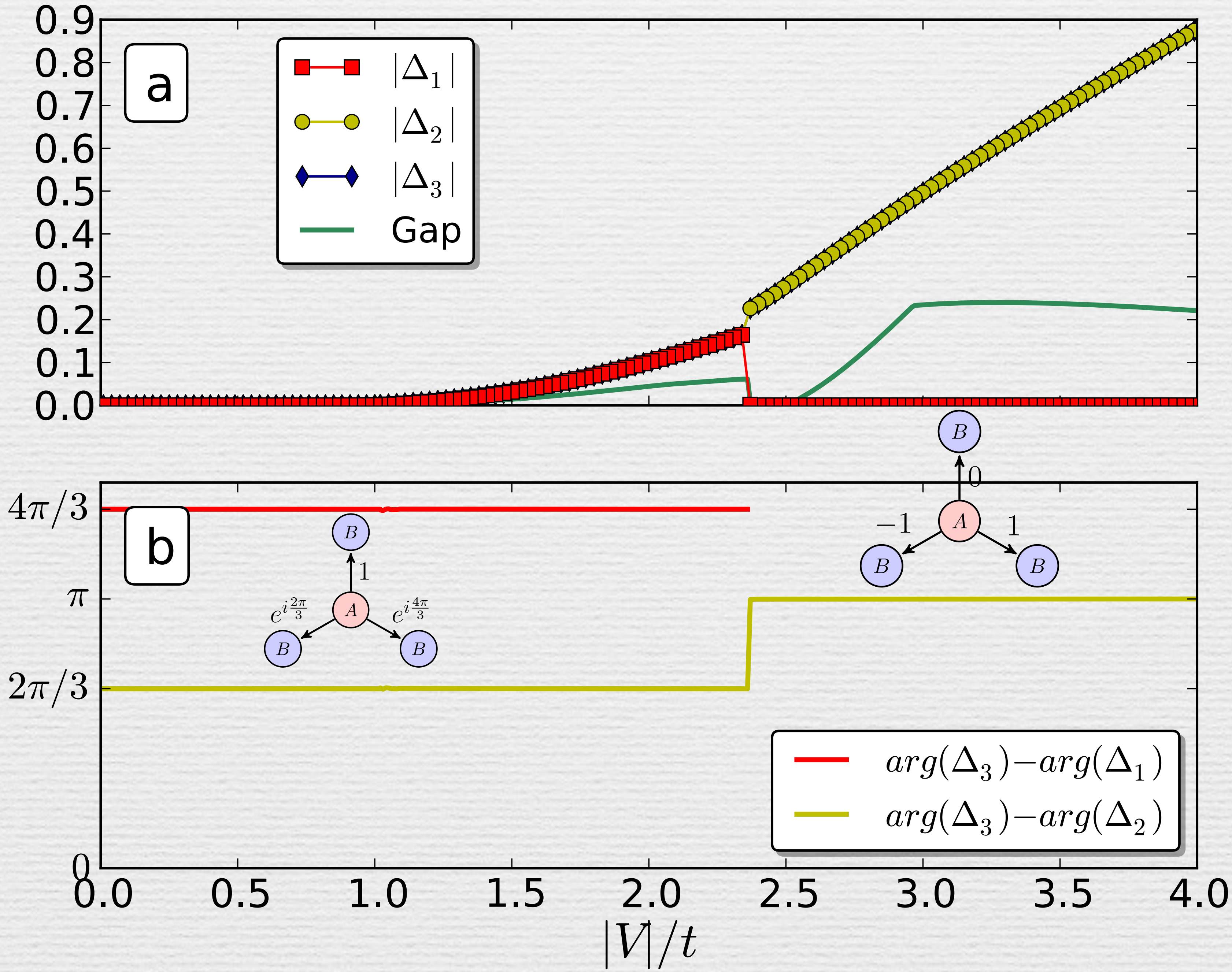
*color indicates size of superconducting gap

Scan V



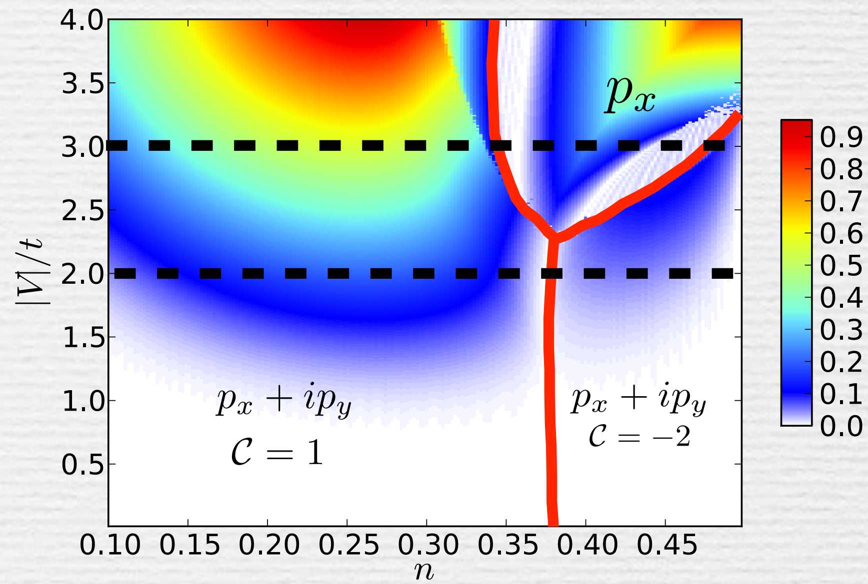
p_x
$p_x + ip_y$
$\mathcal{C} = 1$
$p_x + ip_y$
$\mathcal{C} = -2$
$|V|/t$
n

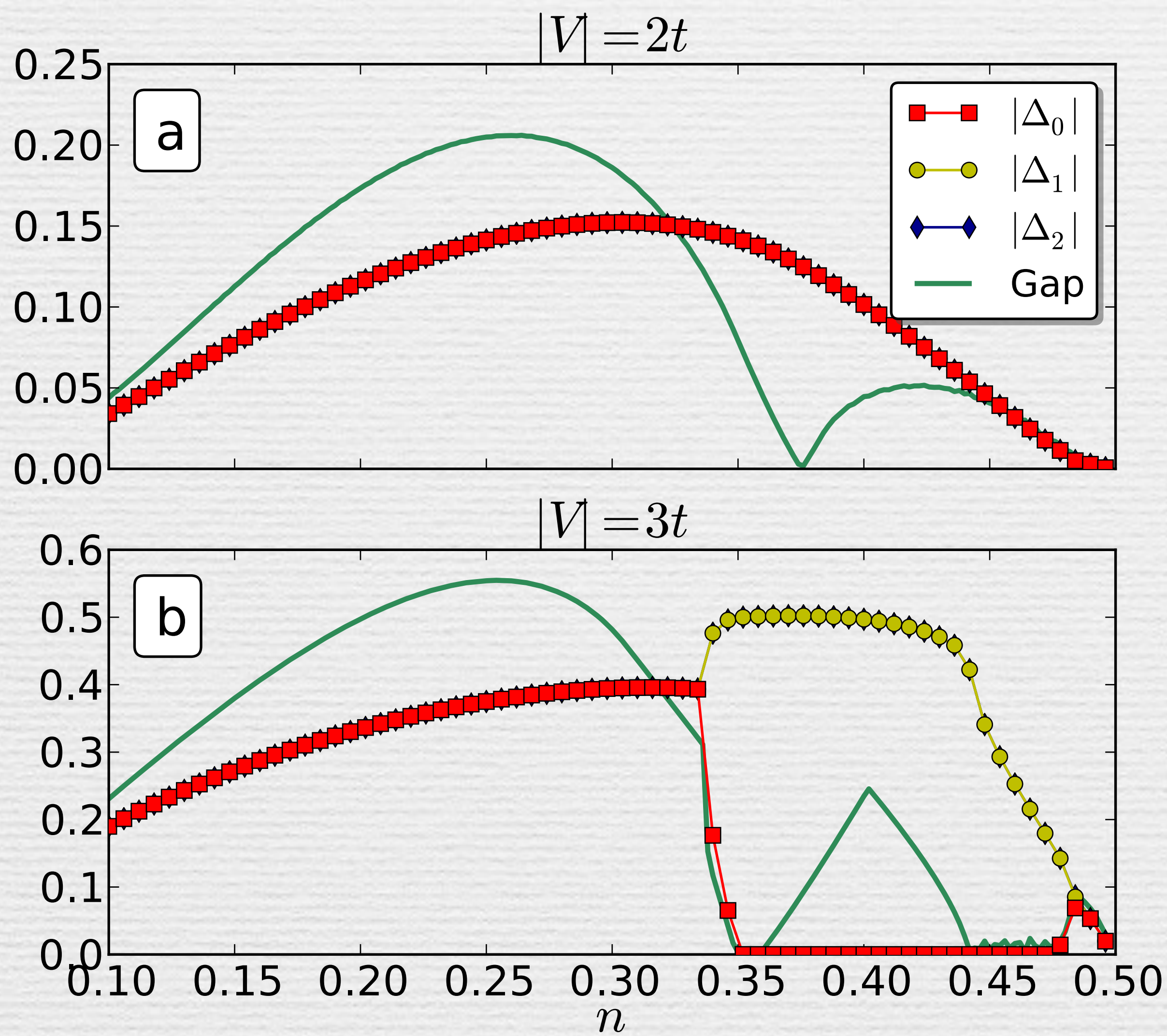
$n = 0.4$



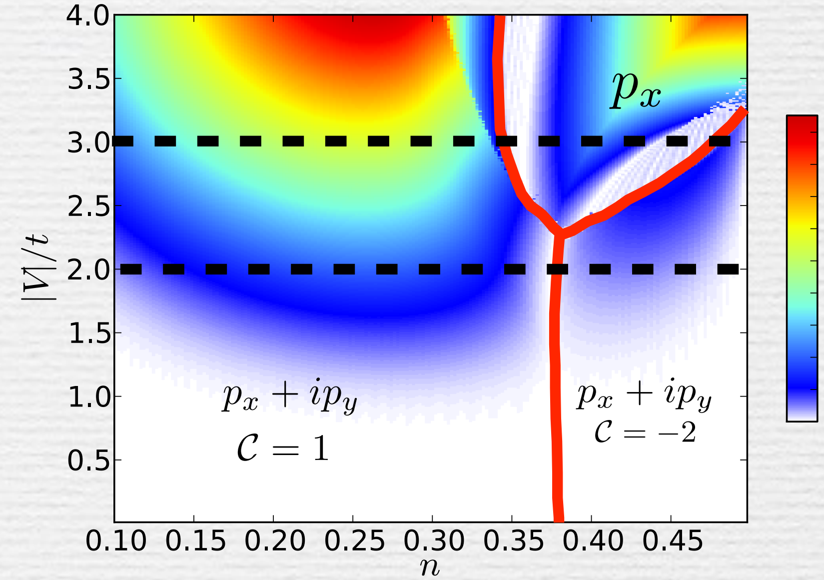
a
$|\Delta_1|$
$|\Delta_2|$
$|\Delta_3|$
Gap
b
$4\pi/3$
π
$2\pi/3$
0
$e^{i\frac{2\pi}{3}}$
$e^{i\frac{4\pi}{3}}$
-1
$arg(\Delta_3) - arg(\Delta_1)$
$arg(\Delta_3) - arg(\Delta_2)$
$|V|/t$

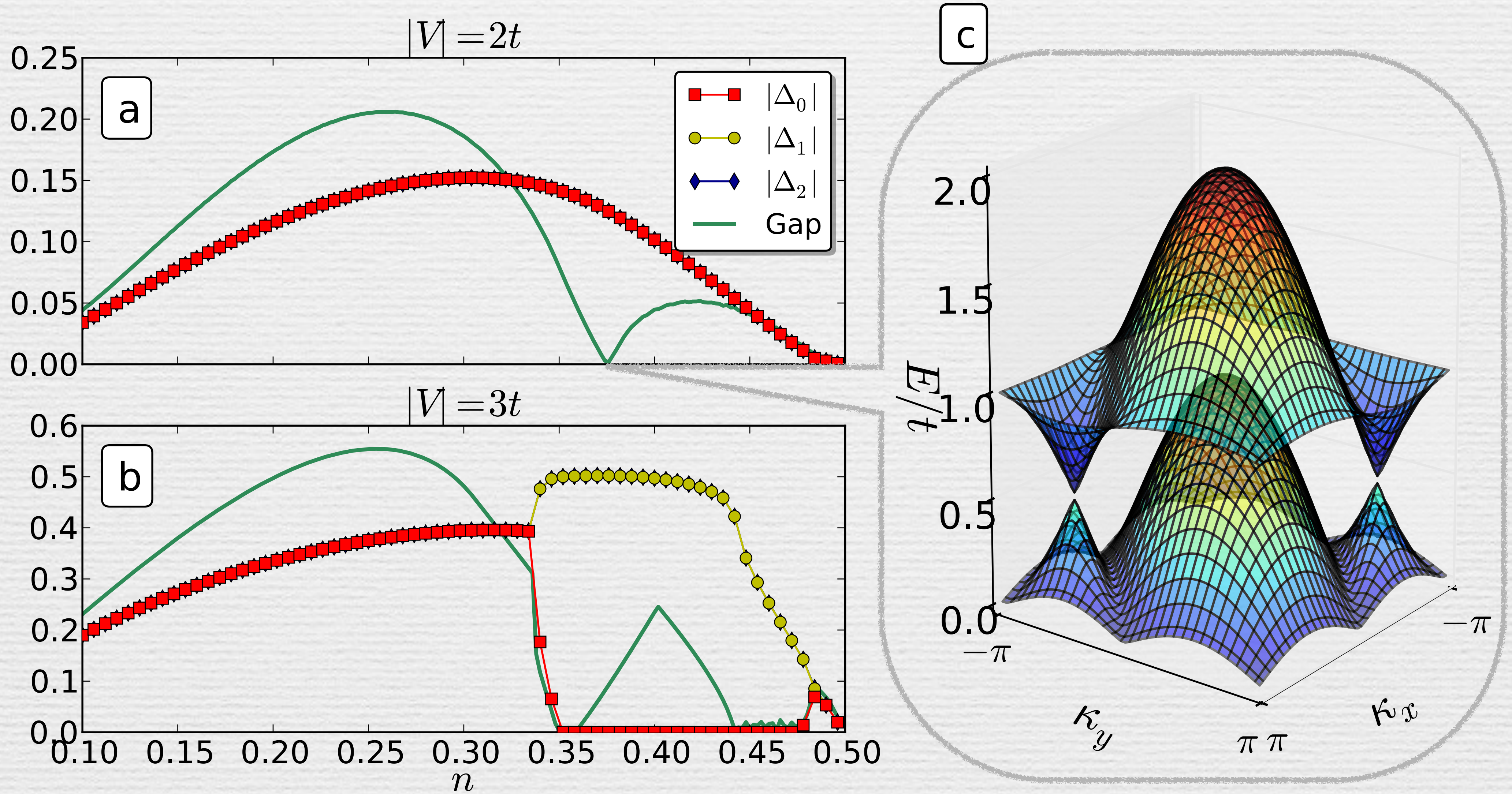
Scan density



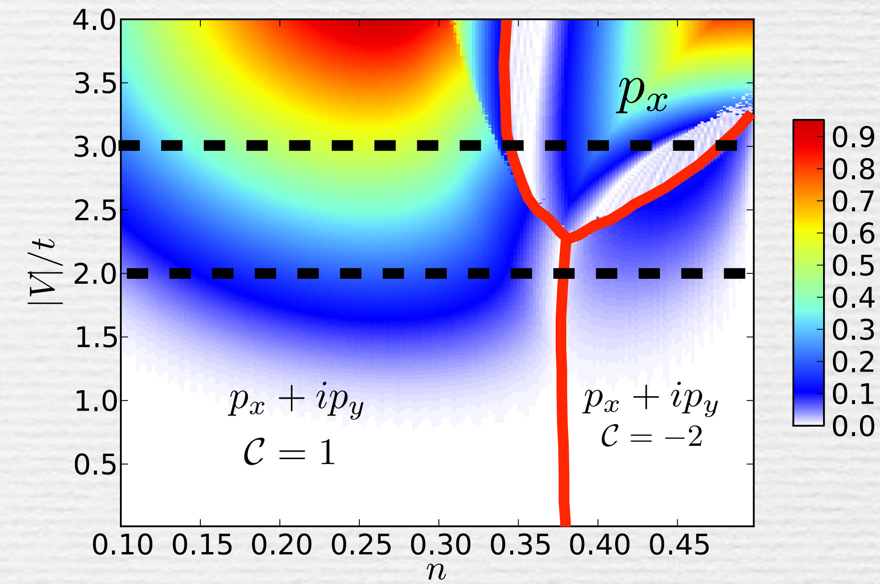


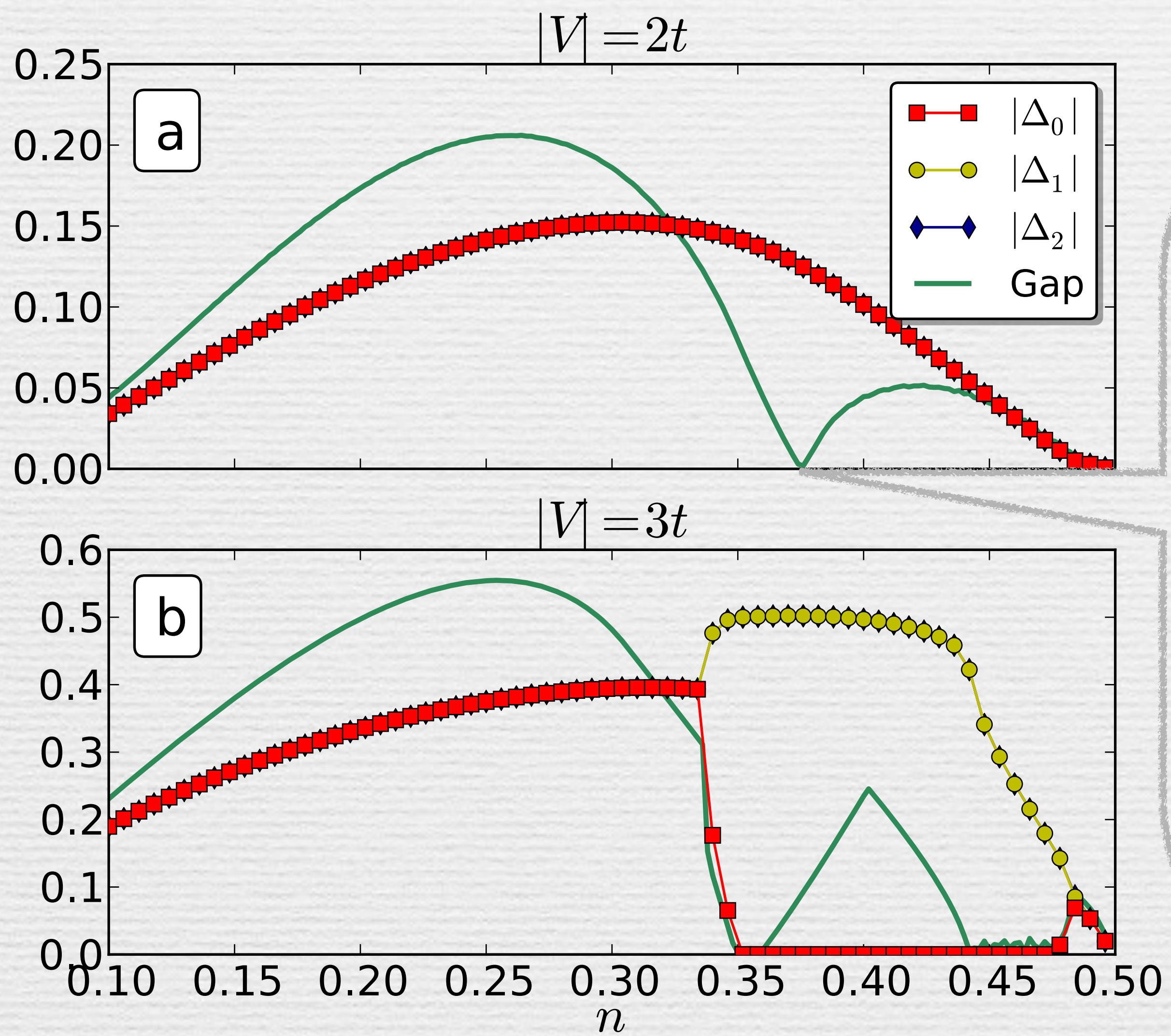
Scan density

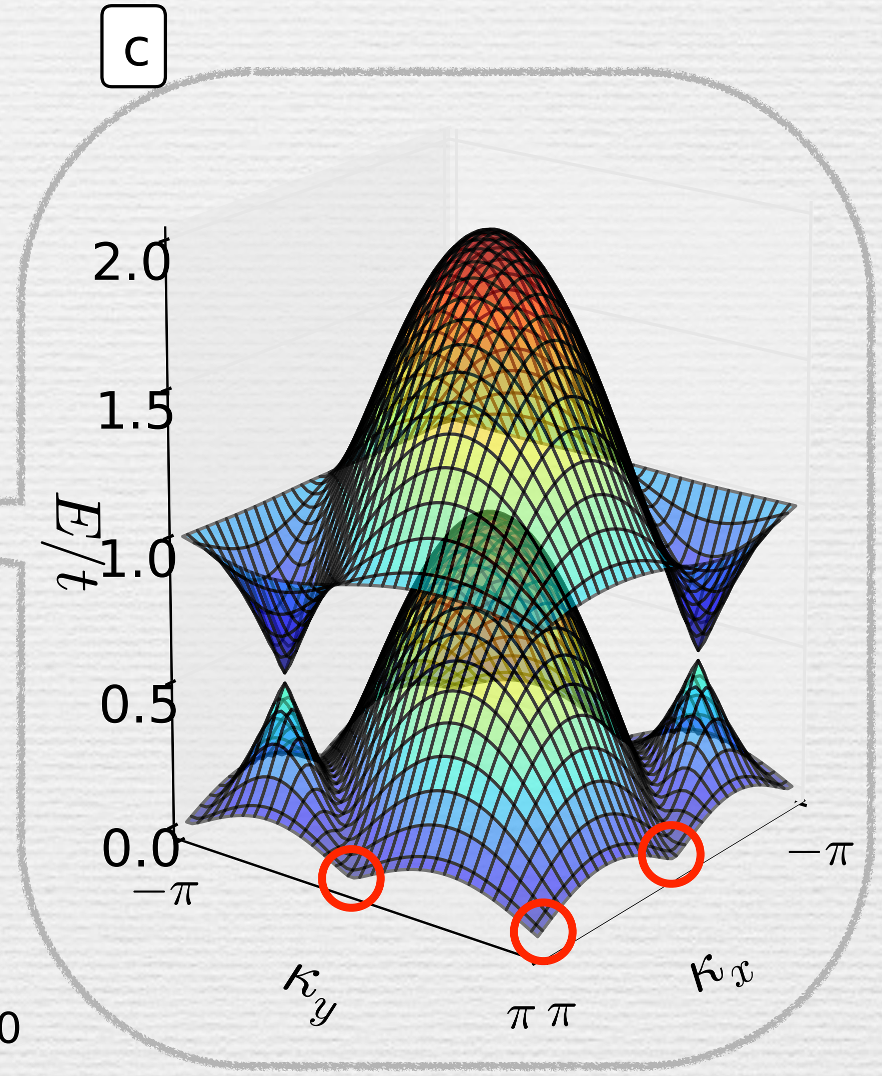




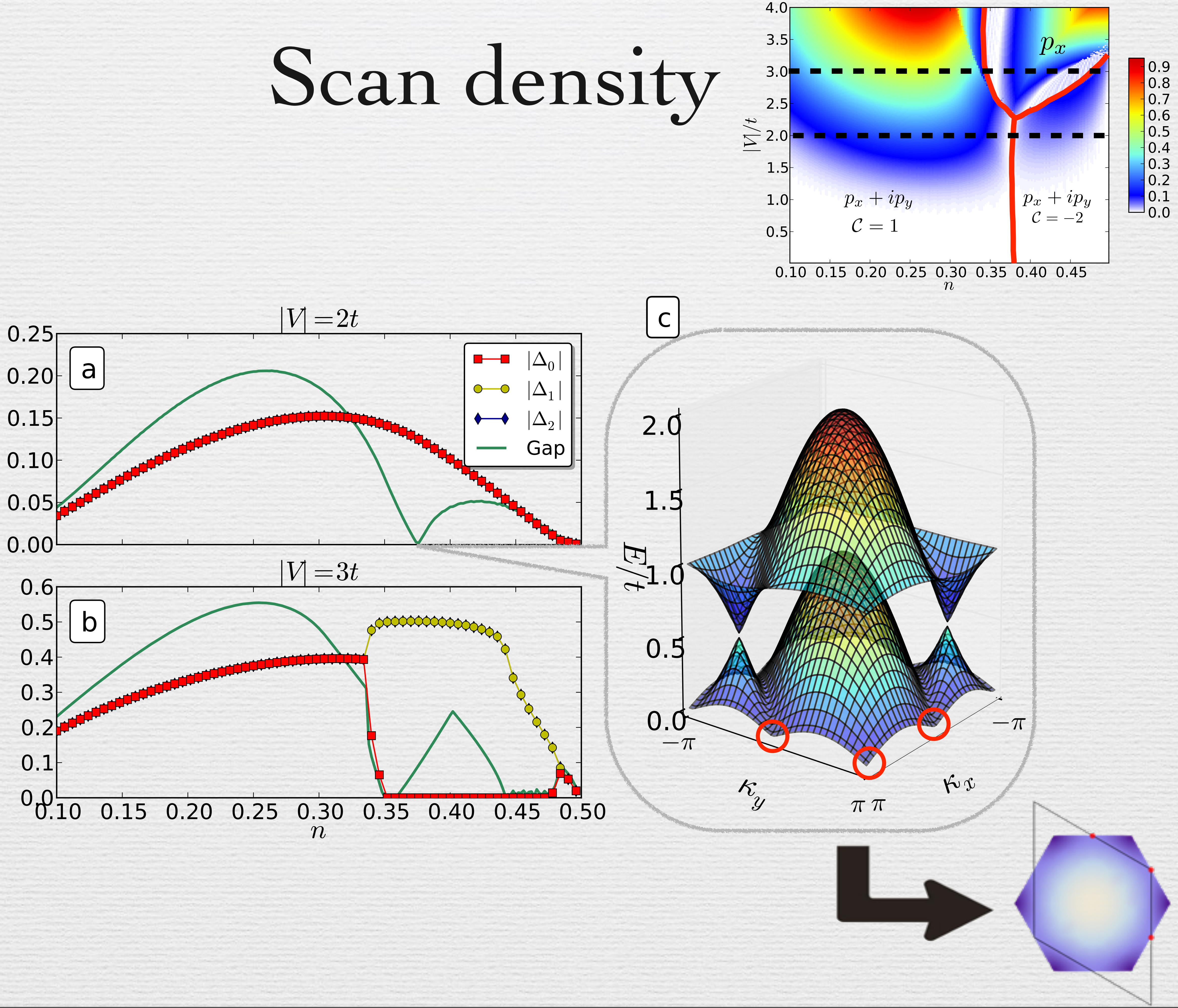
Scan density







Scan density



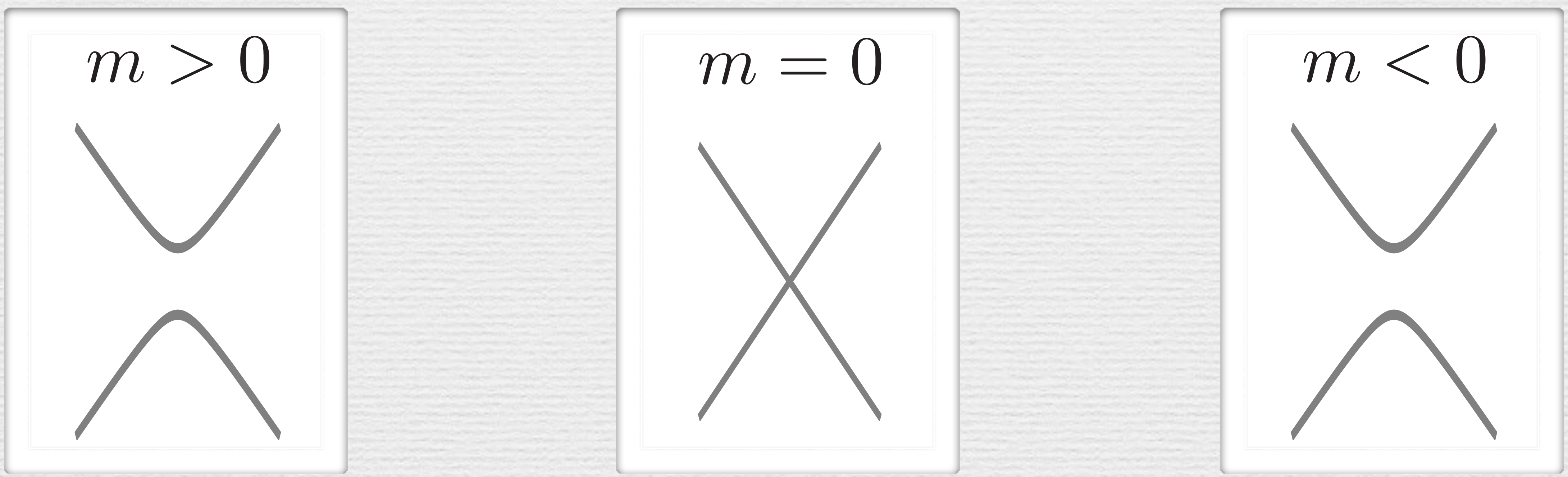
Topological Phase Transition and Dirac Fermions

Chern number of a Dirac fermion

$$\mathcal{C} = \frac{\mathrm{sgn}(m)}{2}$$

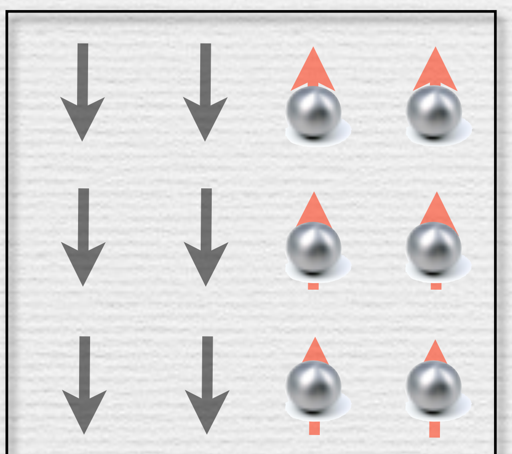
Oshikawa, PRB,1994

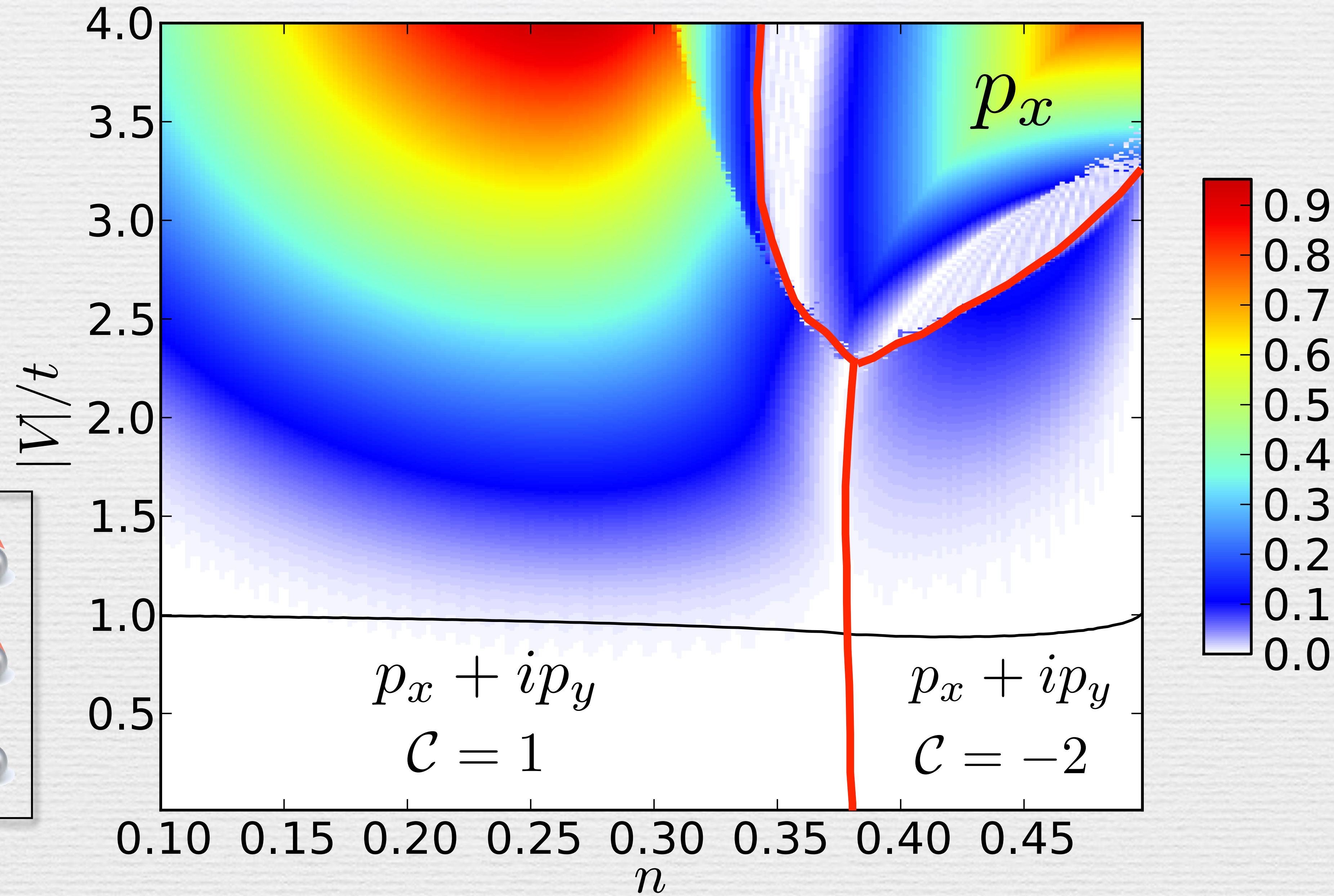
Ludwig et al, PRB,1994



$$\Delta\mathcal{C} = \#\text{ of Dirac points}$$

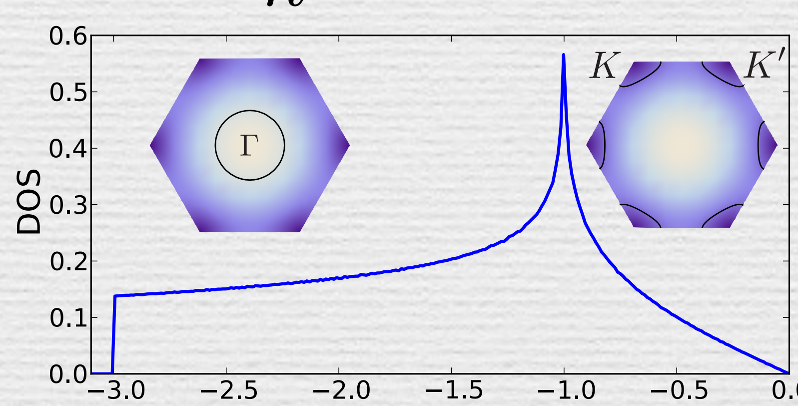
Phase diagram, again





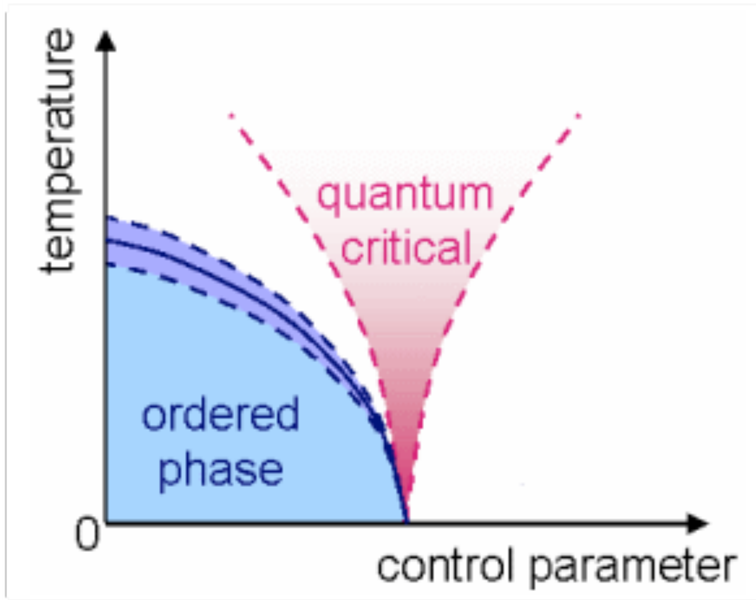
A general "theorem", Cheng, Sun, Galitski and Das Sarma, PRB, 2010

Grassmann Tensor RG, Gu, PRB, 2013

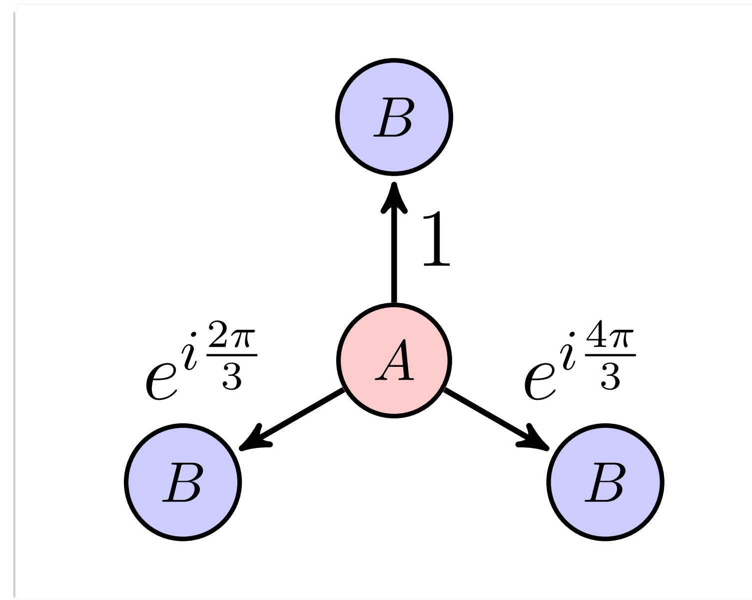


Summary-II

Fermionic
Quantum Critical Point



Topological *Superconductors*



- **Rich physics** in a very simple model
- Hopefully **realizable in experiment** given its simple form

Thank you!